(Section B)

Q. No. 2 (i) (Page 1/2)

$$f(x) = \sqrt{x^3 + 4}$$

As $y = f(x)$ —(i)

$y = \sqrt{x^3 + 4}$

squaring both sides:

$y^2 = x^3 + 4$

$y^2 - 4 = x^3$

$x = (y^2 - 4)^{1/3}$

From(i) $x = f^{-1}(y)$

$f^{-1}(y) = (y^2 - 4)^{1/3}$

replacing x by y

$$f^{-1}(x) = (x^2 - 4)^{1/3}$$

$$f(f^{-1}(x)) = x$$

L.H.S

$= f(f^{-1}(x))$

$= f(x^2 - 4)^{1/3}$

$= \sqrt{[(x^2 - 4)^{1/3}]^3 + 4}$

$= \sqrt{x^2 - 4 + 4}$

$= \sqrt{x^2} = x$

Q. No. 2 (i) (Page 2/2)

Cutting Line

Q. No. 2 (ii) (Page 1/2)

$$\lim_{x \to 0} \frac{\operatorname{cosec} x - \cot x}{x}$$

$$= \lim_{x \to 0} \frac{1}{x} (\operatorname{cosec} x - \cot x)$$

$$= \lim_{x \to 0} \frac{1}{x} \left(\frac{1}{\sin x} - \frac{\cos x}{\sin x} \right) \quad \because \operatorname{cosec} x = \frac{1}{\sin x}$$

$$\because \cot x = \frac{\cos x}{\sin x}$$

$$= \lim_{x \to 0} \frac{1}{x} \left(\frac{1 - \cos x}{\sin x} \right)$$

Multiplying and dividing by $(1 + \cos x)$

$$= \lim_{x \to 0} \frac{1}{x} \left(\frac{1 - \cos x}{\sin x} \cdot \frac{1 + \cos x}{1 + \cos x} \right)$$

$$= \lim_{x \to 0} \frac{1}{x} \left[\frac{1 - \cos^2 x}{\sin x \, (1 + \cos x)} \right]$$

$$= \lim_{x \to 0} \frac{1}{x} \left[\frac{\sin^2 x}{\sin x \, (1 + \cos x)} \right] \quad \because \sin^2 x + \cos^2 x = 1$$

$$= \lim_{x \to 0} \frac{1}{x} \left[\frac{\sin x}{1 + \cos x} \right]$$

$$= \lim_{x \to 0} \left(\frac{\sin x}{x} \right) \cdot \frac{1}{1 + \cos x}$$

$$= \left(\lim_{x \to 0} \frac{\sin x}{x} \right) \left(\lim_{x \to 0} \frac{1}{1 + \cos x} \right)$$

Putting the limits:

$$= (1) \cdot \frac{1}{} \quad \because \lim \frac{\sin \theta}{} = 1$$

Q. No. 2 (ii) (Page 2/2)

$$= \frac{1}{1+1} \quad \because \cos(0) = 1$$

$$= \frac{1}{2}$$

$$\lim_{x \to 0} \frac{\text{cosec}\,x - \cot x}{x} = \frac{1}{2}$$

Cutting Line

Q. No. 2 (iii) (Page 1/2)

$$y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dots \infty}}}$$

Let

$$y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dots \infty}}} \quad \text{---(A)}$$

TO PROVE

$$(2y-1)\frac{dy}{dx} = \cos x$$

Squaring both sides:

$$y^2 = \sin x + \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dots \infty}}}$$

$$y^2 = \sin x + y \quad \because \text{from (A)}$$

$$y^2 - y = \sin x$$

Differentiating both sides with respect to x

$$\frac{d}{dx}(y^2 - y) = \frac{d}{dx}\sin x$$

$$\frac{d}{dx}y^2 - \frac{d}{dx}y = \frac{d}{dx}\sin x$$

$$2y\frac{dy}{dx} - \frac{dy}{dx} = \cos x$$

$$\frac{dy}{dx}(2y-1) = \cos x$$

$$(2y-1)\frac{dy}{dx} = \cos x$$

Hence proved.

Q. No. 2 (iii) (Page 2/2)

Cutting Line

Q. No. 2 (iv) (Page 1/2)

TO PROVE:

$$\sin(x+h) = \sin x + h\cos x - \frac{h^2}{2!}\sin x - \frac{h^3}{3!}\cos x + \cdots \quad -(A)$$

Let Taylor series:

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!}f''(x) + \frac{h^3}{3!}f'''(x) + \cdots \quad -(B)$$

Comparing L.H.S of equation (A) and (B):

$$f(x+h) = \sin(x+h) \quad -(i)$$

OR $$f(x) = \sin x \quad -(ii)$$

For higher derivatives:

$$f'(x) = \cos x \quad -(iii)$$

$$f''(x) = -\sin x \quad -(iv)$$

$$f'''(x) = -\cos x \quad -(V)$$

Putting (i), (ii), (iii), (iv), (v) in equation (B)

$$\sin(x+h) = \sin x + h(\cos x) + \frac{h^2}{2!}(-\sin x) + \frac{h^3}{3!}(-\cos x) + \cdots$$

$$\sin(x+h) = \sin x + h\cos x - \frac{h^2}{2!}\sin x - \frac{h^3}{3!}\cos x + \cdots$$

Hence proved.

Q. No. 2 (iv) (Page 2/2)

Cutting Line

Q. No. 2 (v) (Page 1/2)

$$y = \sin^{-1}\frac{x}{a}$$

TO PROVE
$y_2 = x(a^2 - x^2)^{-3/2}$

Differentiating w.r.t x

$$y_1 = \frac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}}\frac{d}{dx}\left(\frac{x}{a}\right)$$

$\because \frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}\frac{dx}{dx}$

$$y_1 = \frac{1}{\sqrt{\frac{a^2-x^2}{a^2}}}\cdot\frac{1}{a}\frac{dx}{dx}$$

$$= \frac{a}{\sqrt{a^2-x^2}}\cdot\frac{1}{a}(1)$$

$$= \frac{1}{\sqrt{a^2-x^2}}$$

$$y_1 = (a^2-x^2)^{-1/2}$$

Again differentiating w.r.t x

$$y_2 = -\frac{1}{2}(a^2-x^2)^{-\frac{1}{2}-1}(-2x)$$

$$y_2 = x(a^2-x^2)^{-3/2}$$

Hence proved.

Q. No. 2 (v) (Page 2/2)

Cutting Line

Q. No. 2 (vi) (Page 1/2)

$$\int \frac{dx}{3x(\ln 3x)^4} \quad \text{—(A)}$$

Let

$$\ln 3x = t \quad \text{—(i)}$$

Taking differential on both sides:

$$3 \cdot \frac{1}{3x} dx = dt$$

$$\frac{dx}{3x} = \frac{dt}{3} \quad \text{—(ii)}$$

Putting (i) and (ii) in (A)

$$= \int \frac{dt}{3(t)^4}$$

$$= \frac{1}{3} \int (t)^{-4} dt$$

$$= \frac{1}{3} \frac{(t)^{-4+1}}{-4+1} + C$$

$$= \frac{1}{3} \frac{t^{-3}}{-3}$$

$$= \frac{-1}{9t^3} + c$$

$$= \frac{-1}{9(\ln 3x)^3} + C \quad \because \text{from (i)}$$

$$\int \frac{dx}{3x(\ln 3x)^4} = \frac{-1}{9(\ln 3x)^3} + C$$

Q. No. 2 (vi) (Page 2/2)

Q. No. 2 (vii) (Page 1/2)

The relevant question should be answered only in the allotted space and inside the outer mark

Q. No. 2 (vii) (Page 2/2)

Cutting Line

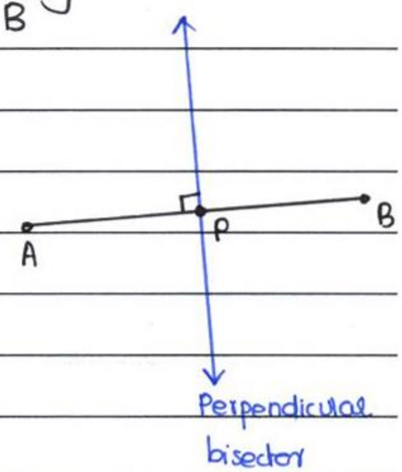
Q. No. 2 (viii) (Page 1/2)

Q. No. 2 (viii) (Page 2/2)

Q. No. 2 (ix) (Page 1/2)

A (5,6) and B (8,4)

⇒ The perpendicular bisector passes through the midpoint of join of A and B points.



Midpoint of A and B:

$x = \frac{x_1 + x_2}{2} = \frac{5+8}{2} = \frac{13}{2}$

$y = \frac{y_1 + y_2}{2} = \frac{6+4}{2} = 5$

$P(x, y) = P\left(\frac{13}{2}, 5\right)$

⇒ The slope of $\overline{AB}$:

$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{6-4}{5-8}$

$= \frac{2}{-3}$

$= \frac{-2}{3}$

Slope of perpendicular bisector:

$m = -\frac{1}{m_{AB}}$

$= -\frac{1}{(-2/3)}$

$m = \frac{3}{}$

Q. No. 2 (ix) (Page 2/2)

⇒ For equation of ⊥ bisector using point slope form

$$y - y_1 = m(x - x_1)$$

where $(x_1, y_1) = \left(\frac{13}{2}, 5\right)$

$$m = \frac{3}{2}$$

$$y - 5 = \frac{3}{2}\left(x - \frac{13}{2}\right)$$

$$y - 5 = \frac{3}{2}\left(\frac{2x - 13}{2}\right)$$

$$4y - 20 = 3(2x - 13)$$

$$4y - 20 = 6x - 39$$

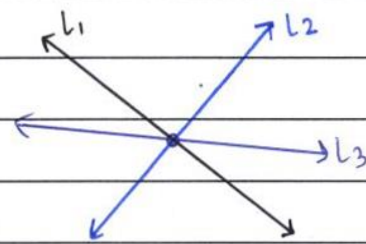
$$6x - 4y - 19 = 0$$

Equation of ⊥ bisector: $6x - 4y - 19 = 0$

Q. No. 2 (x) (Page 1/2)

Lines :

$$2x - 2y + 2 = 0$$
$$3x - 5y - 1 = 0$$
$$2x + ky + 8 = 0$$



The lines meet at a point, i.e, they are **concurrent** so:

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

$$\begin{vmatrix} 2 & -2 & 2 \\ 3 & -5 & -1 \\ 2 & k & 8 \end{vmatrix} = 0$$

$$2\left(-40 + k\right) + 2\left(24 + 2\right) + 2\left(3k + 10\right) = 0$$

$$-80 + 2k + 48 + 4 + 6k + 20 = 0$$
$$8k - 8 = 0$$
$$8k = 8$$
$$k = 1$$

For $\boxed{k = 1}$ lines are concurrent.

Q. No. 2 (x) (Page 2/2)

Cutting Line

Q. No. 2 (xi) (Page 1/2)

$5x + 7y \leq 35$, $-x + 3y \leq 3$

$x \geq 0$, $y \geq 0$

$5x + 7y \leq 35$ —(A)

⇒ Associated equation:

$5x + 7y = 35$ —(i)

⇒ For x-intercept, put $y = 0$ in (i)

$5x + 7(0) = 35$

$x = 7$

$(7,0)$

⇒ For y-intercept, put $x=0$ in (i)

$5(0) + 7y = 35$

$7y = 35$

$y = 5$

$(0,5)$

⇒ For solution region, test a point (0,0) in (A)

$5(0) + 7(0) \leq 35$

$0 \leq 35$

which is TRUE; hence solution region lies towards the origin.

$-x + 3y \leq 3$ —(B)

⇒ Associated equation:

$-x + 3y = 3$ —(ii)

⇒ For x intercept, put $y=0$ in (ii)

9

Q. No. 2 (xi) (Page 2/2)

⇒ For y-intercept, put $x = 0$ in (ii)

$$0 + 3y = 3$$

$$y = 1$$

$$(0, 1)$$

⇒ For solution region, test point $(0,0)$ in (B)

$$0 + 3(0) \leq 3$$

$$0 \leq 3$$

which is TRUE ; solution region lies towards origin.

⇒ For graph, refer page # 59

⇒ The feasible region of solution is shown by shaded area

⇒ Corner points are:

$$(0,0), (0,1), (7,0), (42/11, 25/11)$$

Cutting Line

Q. No. 2 (xii) (Page 1/2)

A $(2,3)$, $B(0,2)$ centre at $3x + 2y - 3 = 0$

let equation of circle

$$(x-h)^2 + (y-k)^2 = r^2 \quad \text{—(A)}$$

⇒ Point A $(2,3)$ lies on circle so it must satisfy circle's equation. Putting A in eq(A)

$$(2-h)^2 + (3-k)^2 = r^2 \quad \text{—(B)}$$

Putting B $(0,2)$ in eq.(A)

$$(0-h)^2 + (2-k)^2 = r^2$$

$$h^2 + (2-k)^2 = r^2 \quad \text{—(C)}$$

⇒ Equating eq.(B) and eq.(C), we get:

$$h^2 + 4 + k^2 - 4k = 4 - 4h + h^2 + 9 + k^2 - 6k$$

$$2k + 4h = 9 \quad \text{—(D)}$$

⇒ Since centre (h,k) of circle lies at $3x + 2y - 3 = 0$, it must satisfy:

$$3h + 2k - 3 = 0$$

$$3h + 2k = 3 \quad \text{—(E)}$$

⇒ Solving (D) and (E) by (D) − (E)

$$4h + 2k = 9$$

$$\pm 3h \pm 2k = \pm 3$$

Q. No. 2 (xii) (Page 2/2)

⇒ Putting value of h in eq (E)

$$3(6) + 2k = 3$$

$$2k = 3 - 18$$

$$k = -\frac{15}{2}$$

⇒ Putting values of h and k in eq (C)

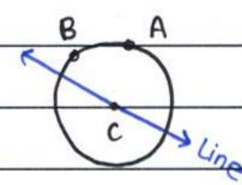
$$(6)^2 + \left(2 + \frac{15}{2}\right)^2 = r^2$$

$$36 + \frac{361}{4} = r^2$$

$$r^2 = \frac{505}{4}$$

Putting values of h, k, r^2 in eq(A) to get equation of circle:

$$(x-6)^2 + \left(y + \frac{15}{2}\right)^2 = \frac{505}{4}$$



Cutting Line

Q. No. 2 (xiii) (Page 1/2)

Focus $(3, 2)$

Directrix : $2x - y + 5 = 0$

let a point $P(x, y)$ lies on parabola,
then by definition:

$$|\overline{PF}| = |\overline{PD}|$$

$\because d = \frac{|ax + by + c|}{\sqrt{a^2 + b^2}}$

$$\sqrt{(x-3)^2 + (y-2)^2} = \frac{|2(x) + (-1)y + 5|}{\sqrt{(2)^2 + (-1)^2}}$$

$$\sqrt{(x-3)^2 + (y-2)^2} = \frac{|2x - y + 5|}{\sqrt{5}}$$

Squaring both sides:

$$5\left[(x-3)^2 + (y-2)^2\right] = (2x - y + 5)^2$$

$$5(x^2 - 6x + 9 + y^2 - 4y + 4) = 4x^2 + y^2 + 25 - 4xy - 10y + 20x$$

$$5x^2 - 30x + 45 + 5y^2 - 20y + 20 = 4x^2 + y^2 + 25 - 4xy - 10y + 20x$$

$$x^2 + 4y^2 - 50x - 10y + 40 + 4xy = 0$$

is the equation of parabola.

Q. No. 2 (xiii) (Page 2/2)

Cutting Line

Q. No. 2 (xiv) (Page 1/2)

Hyperbola: $9x^2 - 4y^2 = 36$ —(A)

Line: $3x + 2y + 7 = 0$

⇒ Dividing eq (A) by 36;

$$\frac{x^2}{4} - \frac{y^2}{9} = 1$$

comparing it with

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

we have:

$a^2 = 4$

$b^2 = 9$

⇒ Equation of tangent line to hyperbola is:

$$y = mx \pm \sqrt{a^2m^2 - b^2} \quad \text{—(B)}$$

⇒ Since tangent line is parallel to given line their slopes are equal, i.e:

$$m_{Tangent} = m_{Line} = \frac{-a}{b}$$

$$m = -\frac{3}{2}$$

⇒ Putting values of m, a^2 and b^2 in eq(B):

$$y = -\frac{3}{2}x \pm \sqrt{4 \cdot \left(-\frac{3}{2}\right)^2 - 9}$$

$$y = -3x \pm \sqrt{4 \cdot 9 - 9}$$

Q. No. 2 (xiv) (Page 2/2)

$2y = -3x$

Equation of tangent line:

$3x + 2y = 0$

Q. No. 2 (xv) (Page 1/2)

Q. No. 2 (xv) (Page 2/2)

Cutting Line

Q. No. 2 (xvi) (Page 1/2)

$\vec{A}$ $(-2, 1, 4)$ $\vec{B}$ $(3, 2, 5)$

$\vec{C}$ $(-3, -5, 0)$ $\vec{D}$ $(5, 8, 9)$

⇒ Volume of tetrahedron = $\frac{1}{6} |\vec{AB} \cdot \vec{AC} \times \vec{AD}|$ —(i)

⇒ For vectors $\vec{AB}$, $\vec{AC}$ and $\vec{AD}$

$\vec{AB}$ = P.V of $\vec{B}$ − P.V of $\vec{A}$
= $(3, 2, 5) - (-2, 1, 4)$
$\vec{AB}$ = $(5, 1, 1)$

$\vec{AC}$ = P.V of $\vec{C}$ − P.V of $\vec{A}$
= $(-3, -5, 0) - (-2, 1, 4)$
= $(-1, -6, -4)$

$\vec{AD}$ = P.V of $\vec{D}$ − P.V of $\vec{A}$
= $(5, 8, 9) - (-2, 1, 4)$
= $(7, 7, 5)$

⇒ $\vec{AB} \cdot \vec{AC} \times \vec{AD} = \begin{vmatrix} 5 & 1 & 1 \\ -1 & -6 & -4 \\ 7 & 7 & 5 \end{vmatrix}$

$= 5(-30 + 28) - 1(-5 + 28) + 1(-7 + 42)$

$= 5(-2) - 1(23) + 1(35)$

Q. No. 2 (xvi) (Page 2/2)

Putting value of in eq(k)

$$Volume = \frac{1}{6}(2)$$

$$= \frac{1}{3}$$

$$Volume = \frac{1}{3} \text{ cubic units}$$

Cutting Line

Q. No. 3 (Page 1/4)

$$f(x) = \begin{cases} mx+3 & , x<3 \\ m+n & , x=3 \\ -x+9 & , x>3 \end{cases}$$

$$\lim_{x \to 3^-} f(x)$$

left hand limit at $x=3$:

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} (mx+3)$$

$$= m(3)+3$$

$$= 3m+3 \quad \text{---(i)}$$

$$\lim_{x \to 3^+} f(x)$$

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (-x+9)$$

$$= -3+9$$

$$= 6 \quad \text{---(ii)}$$

As $f(x)$ is continuous,

Left hand limit = Right hand Limit

$$3m+3 = 6$$

$$3m = 3$$

$$\boxed{m=1}$$

$f(3) = m+n$

$f(3) = 1+5$

$= 6$

(i) ⇒ $\lim_{x\to 3^-} f(x) = 3(1)+3$

$= 6$

Q. No. 3 (Page 2/4)

$f(3) = \lim_{x \to 3} f(x)$

$m + n = 6$

$1 + n = 6$

$n = 5$

GRAPH:

$f(x) = mx + 3, \quad m + n, \quad -x + 9$

x	0	1	2	3	4	5
y				6		

Cutting Line

Q. No. 3 (Page 3/4)

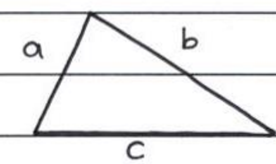
The relevant question should be answered only in the allotted space and inside the outer mark

Q. No. 3 (Page 4/4)

Cutting Line

Q. No. 4 (Page 1/4)

→ Let three sides of a triangle are a, b, c, then:



$a = 8$

$b = x$

$c = \text{Perimeter} - a - b$

$= 18 - 8 - x$

$c = 10 - x$

→ The value of "s" is:

$\frac{a+b+c}{2} = s$

$s = \frac{18}{2}$

$s = 9$

(a) f(x) is the area to be maximized

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{9(9-8)(9-x)(9-10+x)}$

$A = \sqrt{9(9-x)x-1)}$

squaring both sides:

$A^2 = 9(9-x)(x-1)$

$f(x) = 9(9-x)(x-1)$

$= 9(9x - 9 - x^2 + x)$

Q. No. 4 (Page 2/4)

(b)

For $f'(x)$ differentiating both sides of (K) w.r.t x

$f'(x) = \frac{d}{dx}(90x - 81 - 9x^2)$

$f'(x) = 90 - 18x$ —(L)

For $f''(x)$, again differentiating

$f''(x) = -18x$ —(M)

(c)

Putting $f'(x) = 0$ in eq. (L)

$0 = 90 - 18x$

$18x = 90$

$x = \frac{90}{18}$

$x = 5$

Putting $x = 5$ in (M)

$f''(5) = -18(5)$

$= -90$ (-ve)

which shows $f(x)$ has **maximum value** at $x = 5$

Cutting Line

Q. No. 4 (Page 3/4)

(d)

$a = 8$

$b = x = 5$

$c = 10 - x$

$= 10 - 5$

$= 5$

$a = 8$

$b = 5$

$c = 5$

Q. No. 4 (Page 4/4)

Cutting Line

Q. No. 5 (Page 1/4)

Q. No. 5 (Page 2/4)

Cutting Line

Q. No. 5 (Page 3/4)

Q. No. 5 (Page 4/4)

Cutting Line

Q. No. 6 (Page 1/4)

A $(-2,3)$ B $(4,5)$ C $(6,2)$

(a) $m = y_2 - y_1 / x_2 - x_1$

Slope of side $\overline{AB}$

$$m_{AB} = \frac{5-3}{4-(-2)}$$

$$= \frac{2}{4+2}$$

$$m_{AB} = \frac{1}{3}$$

Slope of side $\overline{BC}$

$$m_{BC} = \frac{2-5}{6-4}$$

$$m_{BC} = \frac{-3}{2}$$

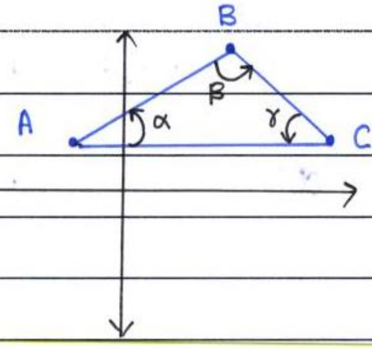
Slope of side $\overline{AC}$

$$m_{AC} = \frac{2-3}{6+2}$$

$$m_{AC} = -\frac{1}{8}$$

Q. No. 6 (Page 2/4)

(b) ⇒ Angle between sides $\overline{AB}$ and $\overline{BC}$



$$\tan\beta = \frac{m_{BC} - m_{AB}}{1 + m_{BC}m_{AB}}$$

$$= \frac{(-3/2) - (1/3)}{1 + (-\frac{3}{2})(\frac{1}{3})}$$

$$= \frac{-11/6}{1/2}$$

$$\tan\beta = \frac{-11}{3}$$

$$\beta = \tan^{-1}\left(-\frac{11}{3}\right)$$

$$\beta = -74.74°$$

$$= -74.74° + 180°$$

$$\boxed{\beta = 105.26°}$$

⇒ Angle between sides $\overline{AB}$ and $\overline{AC}$

$$\tan\alpha = \frac{m_{AB} - m_{AC}}{1 + m_{AB}m_{AC}}$$

$$= \frac{(1/3) - (-1/8)}{1 + (1/3)(-1/8)}$$

$$= \frac{11/24}{23/24}$$

$$\tan\alpha = \frac{11}{23}$$

$$\alpha = \tan^{-1}\left(\frac{11}{23}\right)$$

Cutting Line

Q. No. 6 (Page 3/4)

(C)

⇒ Equation of side $\overline{AB}$:

by point slope form:

$$y - y_1 = m(x - x_1)$$

$$m_{AB} = \frac{1}{3}, \quad (x_1, y_1) = (4, 5)$$

$$y - 5 = \frac{1}{3}(x - 4)$$

$$3y - 15 = x - 4$$

$$\boxed{x - 3y + 11 = 0}$$

⇒ Equation of side $\overline{BC}$:

using point slope form:

$$y - y_1 = m(x - x_1)$$

$$m_{BC} = \frac{-3}{2}, \quad (x_1, y_1) = (4, 5)$$

$$y - 5 = -\frac{3}{2}(x - 4)$$

$$2y - 10 = -3x + 12$$

$$\boxed{3x + 2y - 22 = 0}$$

9

Q. No. 6 (Page 4/4)

(d) Area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} -2 & 3 & 1 \\ 4 & 5 & 1 \\ 6 & 2 & 1 \end{vmatrix}$

$= \frac{1}{2}\left[-2(5-2) - 3(4-6) + 1(8-30)\right]$

$= \frac{1}{2}\left[-2(3) - 3(-2) + 1(-22)\right]$

$= \frac{1}{2}(-6 + 6 - 22)$

Area $= -11$

(neglecting negative sign)

Area = 11 square units

Since $A \neq 0$, points A, B, C are not collinear

Cutting Line

Q. No. 7 (Page 1/4)

⇒ Let x represent chairs and y represent tables.

⇒ The total space is for 28 items, i.e:

$$x + y \leq 28$$

⇒ The cost and total investment subjects:

$$480x + 300y \leq 12000$$

⇒ The profit function which is to be maximized is:

$$f(x,y) = 200x + 150y \quad \text{—(K)}$$

⇒ The constraints are:

$$x + y \leq 28$$

$$480x + 300y \leq 12000$$

Since x and y can't be negative so

$$x \geq 0, \; y \geq 0$$

$$x + y \leq 28 \quad \text{—(i)}$$

⇒ Associated equation:

$$x + y = 28 \quad \text{—(ii)}$$

⇒ For x-intercept, put y = 0 in (ii)

$$x + 0 = 28$$

$$x = 28$$

$$(28, 0)$$

Q. No. 7 (Page 2/4)

⇒ For y-intercept, putting $x = 0$ in (ii)

$0 + y = 28$

$y = 28$

$(0, 28)$

⇒ For solution region, test $(0,0)$ in (i)

$0 + 0 \leq 28$

$0 \leq 28$

which is TRUE, hence solution region lies towards origin.

$480x + 300y \leq 12000$ —(iii)

⇒ Associated equation:

$480x + 300y = 12000$ —(iv)

⇒ For x-intercept, put $y=0$ in (iv)

$480x + 300(0) = 12000$

$x = 25$

$(25, 0)$

⇒ For y-intercept, put $x = 0$ in (iv)

$480(0) + 300y = 12000$

$y = 40$

$(0, 40)$

⇒ For solution region, test $(0,0)$ in (iii)

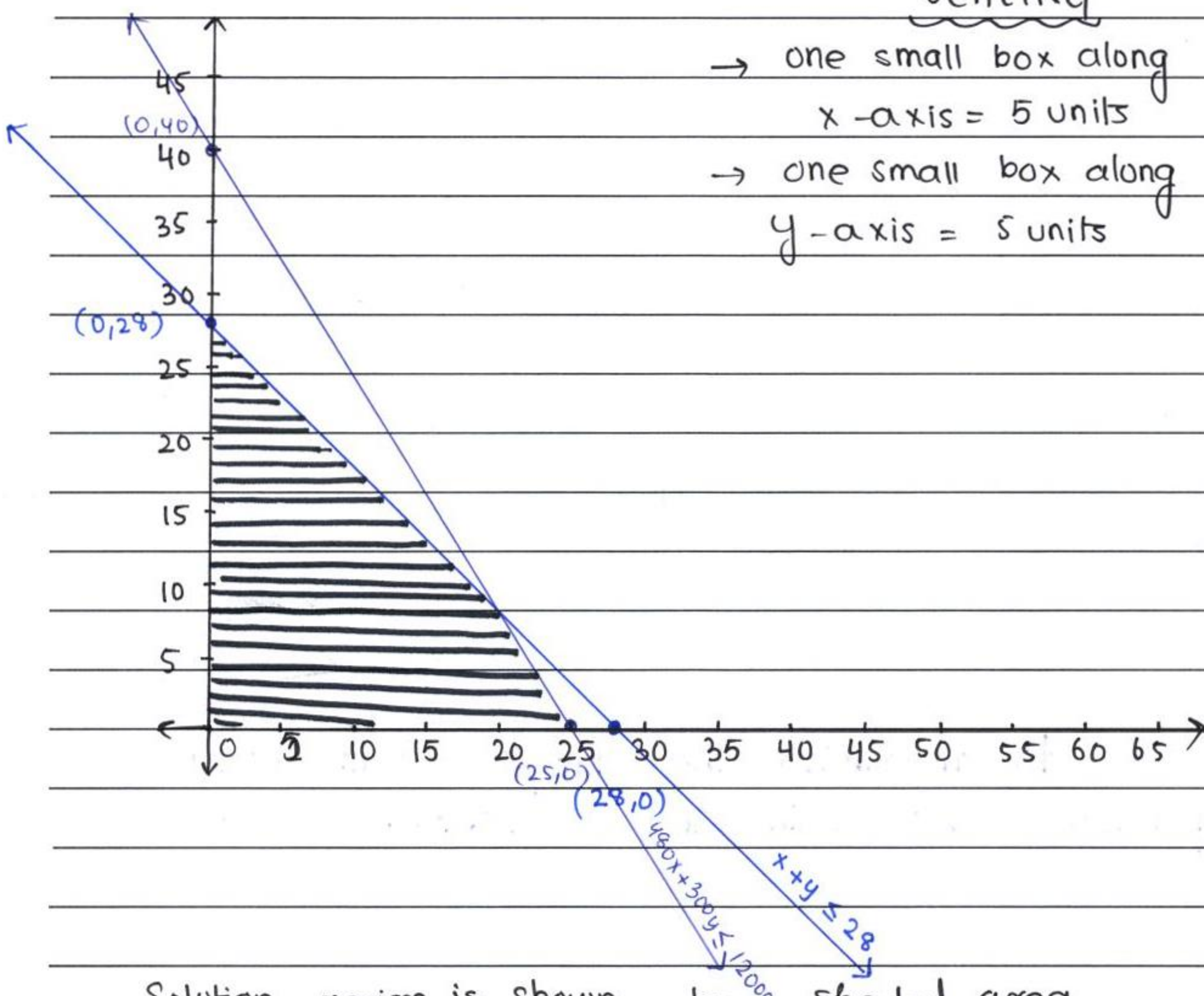
$480(0) + 300(0) \leq 12000$

Q. No. 7 (Page 3/4)

SCALING

→ one small box along x-axis = 5 units

→ one small box along y-axis = 5 units



Solution region is shown by shaded area.

→ Corner points are:

A (0,0), B (25,0), C (0,28) and D (x,y)

For D (x,y) solving (ii) and (iv) eq:

(ii) × 300 − (iv)

$$300x + 300y = 8400$$

$$\pm 480x \pm 300y = \pm 12000$$

$$-180x = -3600$$

$$x = 20$$

Q. No. 7 (Page 4/4)

A (0, 0) B(25, 0) C (0, 28) D (20, 8)

⇒ To maximize the profit putting corner points in (K)

$f(x,y) = 200x + 150y$

$A(0,0) \Rightarrow f(0,0) = 200(0) + 150(0) = 0$

$B(25,0) \Rightarrow f(25,0) = 200(25) + 150(0)$

$= 5000$

$C(0,28) \Rightarrow f(0,28) = 200(0) + 150(28)$

$= 4200$

$D(20,8) \Rightarrow f(20,8) = 200(20) + 150(8)$

$= 5200$

At (20,8) f(x,y) gives maximum value thus to maximize the profit agent should purchase 20 chairs and 8 tables.

Q. No. 8 (Page 1/4)

$$25x^2 + 4y^2 - 250x - 16y + 541 = 0$$

$\Rightarrow$ completing square:

$$25(x^2 - 10x + 25) + 4(y^2 - 4y + 4) = -541 + 625 + 16$$

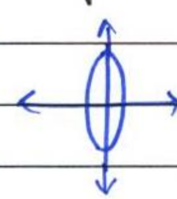
$$25(x-5)^2 + 4(y-2)^2 = 100$$

Dividing 100 on both sides:

$$\frac{25}{100}(x-5)^2 + \frac{4}{100}(y-2)^2 = \frac{100}{100}$$

$$\frac{(x-5)^2}{4} + \frac{(y-2)^2}{25} = 1 \quad -(A)$$

The equation is of an ellipse along y - axis.



Let

$$x - 5 = X \quad -(i)$$

$$y - 2 = Y \quad -(ii)$$

$$(A) \Rightarrow \frac{Y^2}{25} + \frac{X^2}{4} = 1$$

comparing it with general equation of ellipse:

$$\frac{Y^2}{a^2} + \frac{X^2}{b^2} = 1$$

$a^2 = 25$, $b^2 = 4$

Q. No. 8 (Page 2/4)

⇒ In ellipse;

$a^2 = b^2 + c^2$

$25 = 4 + c^2$

$c^2 = 21$

$c = \sqrt{21}$

⇒ The Centre in translated form is

$O'(0,0)$

$X = 0$, $Y = 0$

from (i) & (ii):

$x - 5 = 0$, $y - 2 = 0$

$x = 5$, $y = 2$

Centre (5, 2)

⇒ In translated form, foci are:

$f(0, c)$, $f'(0, -c)$

$X = 0$, $Y = c$ $X = 0$ $Y = -c$

putting (i) and (ii)

$x - 5 = 0$, $y - 2 = \sqrt{21}$, $x - 5 = 0$, $y - 2 = -\sqrt{21}$

$x = 5$, $y = \sqrt{21} + 2$, $x = 5$, $y = -\sqrt{21} + 2$

$F(5, \sqrt{21} + 2)$ and $F'(5, -\sqrt{21} + 2)$

Cutting Line

Q. No. 8 (Page 3/4)

⇒ Eccentricity is:

$$e = \frac{c}{a}$$

$$e = \frac{\sqrt{21}}{5}$$

$$e = \frac{\sqrt{21}}{5}$$

⇒ Vertices of ellipse are:

$V(0, a)$, $V'(0, -a)$

$X = a$, $Y = a$, $X = 0$, $Y = -a$

$x - 5 = 0$, $y - 2 = 5$, $x - 5 = 0$, $y - 2 = -5$

$x = 05$, $y = 07$, $x = 5$, $y = -3$

$V(5, 7)$ and $V'(5, -3)$

⇒ Equation of directrix:

$$y = \pm \frac{a^2}{c}$$

$$Y = \pm \frac{a^2}{c}$$

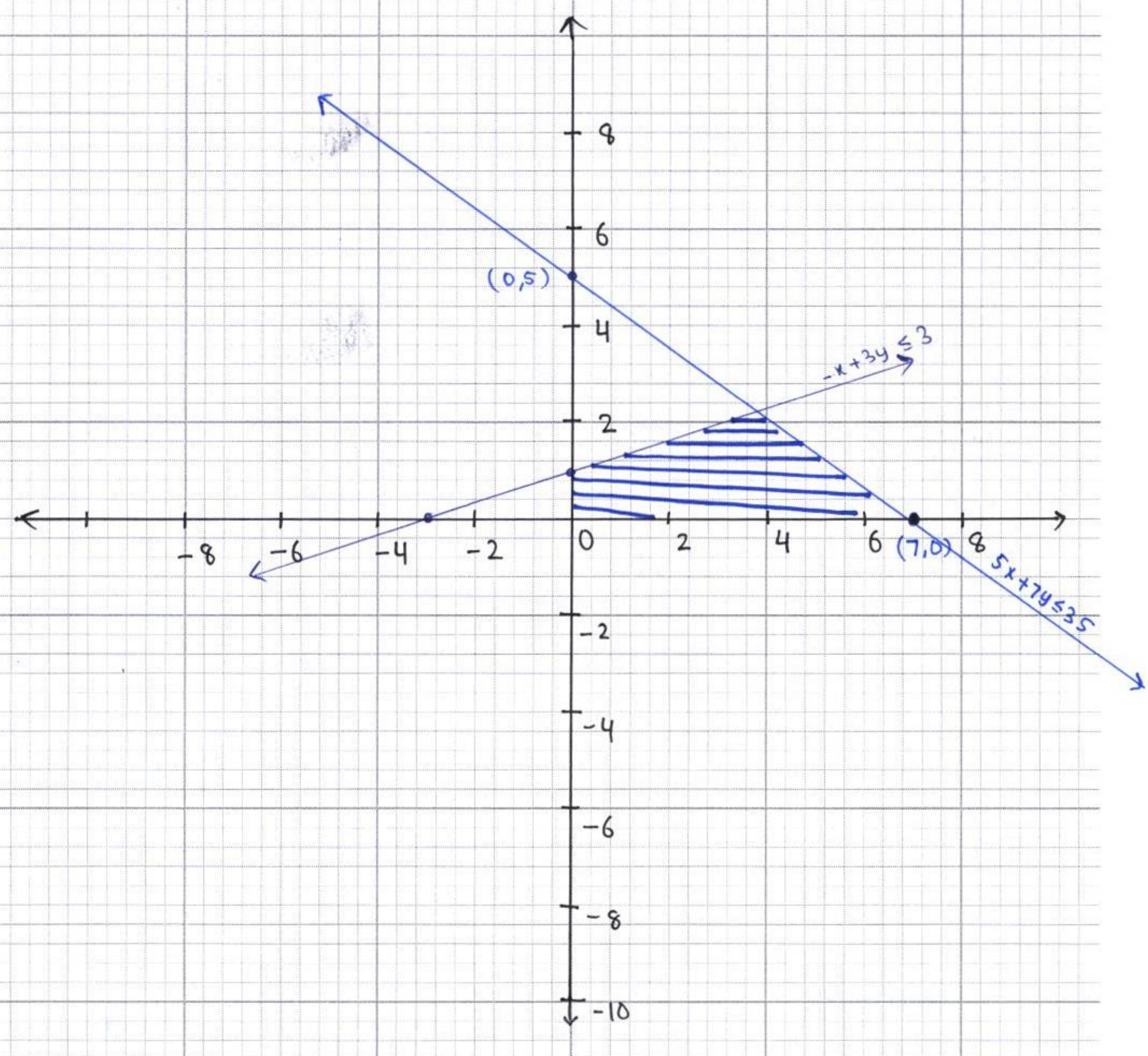
$$y - 2 = \pm \frac{25}{\sqrt{21}}$$

$$y = \pm \frac{25}{\sqrt{21}} + 2$$

Q. No. 8 (Page 4/4)

Cutting Line

Q2 (xi)



SCALING

→ five small boxes on x-axis = 2 units

Graph Page No. 2

Cutting Line