(Section B)

Q. No. 2 (i) (Page 1/2)

Finding $f^{-1}(x)$

The given function is ; $f(x) = \sqrt{x^3+4}$

Let $y = f(x) = \sqrt{x^3+4}$ —①

Then ; $f^{-1}(y) = x$ —②

Taking ① ;

$$y = \sqrt{x^3+4}$$
$$y = (x^3+4)^{1/2}$$

Taking square on both sides;

$$y^2 = x^3+4$$
$$y^2-4 = x^3$$

Taking cube root on both sides;

$$\sqrt[3]{y^2-4} = \sqrt[3]{x^3}$$
$$(y^2-4)^{1/3} = x$$

From ② ;

$$f^{-1}(y) = (y^2-4)^{1/3}$$

Replacing y by x;

$$\mathbf{f^{-1}(x) = (x^2-4)^{1/3}}$$

Verification;

Taking L.H.S ; $f(f^{-1}(x))$

$$= f\left((x^2-4)^{1/3}\right)$$
$$= \sqrt{\left((x^2-4)^{\cancel{1/3}}\right)^{\cancel{3}}+4}$$
$$= \sqrt{x^2-4+4}$$
$$= \sqrt{x^2}$$
$$= x$$
$$= R.H.S$$

Hence, it is proved that $f(f^{-1}(x)) = x$.

Q. No. 2 (i) (Page 2/2)

Q. No. 2 (ii) (Page 1/2)

$$\lim_{x\to 0} \frac{\operatorname{cosec} x - \cot x}{x}$$

$$= \lim_{x\to 0} \frac{\frac{1}{\sin x} - \frac{\cos x}{\sin x}}{x}$$

$$= \lim_{x\to 0} \frac{\frac{1-\cos x}{\sin x}}{x}$$

$$= \lim_{x\to 0} \frac{\sin x \left(\frac{1-\cos x}{\sin^2 x}\right)}{x}$$

$$= \lim_{x\to 0} \frac{\sin x \left(\frac{1-\cos x}{1-\cos^2 x}\right)}{x} \qquad (\because \sin^2 x = 1-\cos^2 x)$$

$$= \lim_{x\to 0} \frac{\sin x \left(\frac{\cancel{(1-\cos x)}}{\cancel{(1-\cos x)}(1+\cos x)}\right)}{x}$$

$$= \lim_{x\to 0} \frac{\sin x \left(\frac{1}{1+\cos x}\right)}{x}$$

$$= \lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{1}{1+\cos x}$$

$$= \lim_{x\to 0} \frac{\sin x}{x} \cdot \lim_{x\to 0} \frac{1}{1+\cos x}$$

$$= 1 \cdot \frac{1}{1+\cos 0} \qquad \left(\because \lim_{x\to 0} \frac{\sin x}{x} = 1\right)$$

Q. No. 2 (ii) (Page 2/2)

$= \frac{1}{2}$

So, $\lim_{x \to 0} \frac{\operatorname{cosec} x - \cot x}{x} = \frac{1}{2}$

Q. No. 2 (iii) (Page 1/2)

$$y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots \infty}}} \quad -(1)$$

Sqquaring both sides of eqquation;

$$y^2 = \sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots \infty}}$$

$$y^2 = \sin x + y$$

$$y^2 - y = \sin x$$

Taking derivative w.r.t 'x';

$$\frac{d}{dx}(y^2 - y) = \frac{d}{dx}(\sin x)$$

$$2y\frac{dy}{dx} - \frac{dy}{dx} = \cos x$$

$$(2y-1)\frac{dy}{dx} = \cos x$$

This is the reqquired form. Hence, it is proved that $(2y-1)\frac{dy}{dx} = \cos x$

Q. No. 2 (iii) (Page 2/2)

Q. No. 2 (iv) (Page 1/2)

$$\sin(x+h) = \sin x + h\cos x - \frac{h^2}{2!}\sin x - \frac{h^3}{3!}\cos x + \dots \quad (1)$$

From the given equation;

$f(x+h) = \sin(x+h)$

$\Rightarrow f(x) = \sin x \quad (2)$

Taking derivatives of eq. (2) w.r.t x;

$f'(x) = \cos x$

$f''(x) = -\sin x$

$f'''(x) = -\cos x$

$\vdots$

The Taylor's series is given as follows;

$$f(x+h) = f(x) + f'(x)h + \frac{f''(x)}{2!}h^2 + \frac{f'''(x)}{3!}h^3 + \dots \quad (3)$$

Putting the acquired values of derivatives in eq (3);

$$\sin(x+h) = \sin x + (\cos x)h + \frac{(-\sin x)}{2!}h^2 + \frac{(-\cos x)}{3!}h^3 + \dots$$

$$\sin(x+h) = \sin x + h\cos x - \frac{h^2}{2!}\sin x - \frac{h^3}{3!}\cos x + \dots$$

Hence, proved

Q. No. 2 (iv) (Page 2/2)

Q. No. 2 (v) (Page 1/2)

$$y = \sin^{-1}\frac{x}{a} \quad -(1)$$

To Show: $y_2 = x\,(a^2-x^2)^{-\frac{3}{2}}$

Differentiating eq (1) w.r.t x;

$$\frac{d}{dx}(y) = \frac{d}{dx}\left(\sin^{-1}\frac{x}{a}\right)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}} \cdot \frac{d}{dx}\left(\frac{x}{a}\right) \qquad \because \sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{\frac{a^2-x^2}{a^2}}} \cdot \frac{1}{a}$$

$$= \frac{1}{\frac{\sqrt{a^2-x^2}}{a}} \cdot \frac{1}{a}$$

$$\frac{dy}{dx} = \frac{\cancel{a}}{\sqrt{a^2-x^2}} \cdot \frac{1}{\cancel{a}}$$

$$y_1 = \frac{dy}{dx} = \frac{1}{\sqrt{a^2-x^2}}$$

$$y_1 = (a^2-x^2)^{-\frac{1}{2}} \quad -(2)$$

Differentiating eq. (2) w.r.t x;

$$\frac{d}{dx}(y_1) = \frac{d}{dx}\left((a^2-x^2)^{-\frac{1}{2}}\right)$$

Q. No. 2 (v) (Page 2/2)

$$y_2 = \frac{-1}{\cancel{2}}(a^2-x^2)^{\frac{-3}{2}} \cdot (-\cancel{2}x)$$

$$\boxed{y_2 = x(a^2-x^2)^{\frac{-3}{2}}}$$

Hence, proved.

Q. No. 2 (vi) (Page 1/2)

$$\int \frac{dx}{3x(\ln 3x)^4}$$

$$= \int (\ln 3x)^{-4} \cdot \frac{1}{3x} \, dx$$

$$= \frac{1}{3} \int (\ln 3x)^{-4} \cdot \frac{1}{3x} \cdot 3 \, dx$$

$$= \frac{(\ln 3x)^{-4+1}}{-4+1} + c$$

$$= \frac{(\ln 3x)^{-3}}{-3} + c$$

$$= \frac{-1}{3(\ln 3x)^3} + c$$

So, $\displaystyle\int \frac{dx}{3x(\ln 3x)^4} = \frac{-1}{3(\ln 3x)^3} + c$

Q. No. 2 (vi) (Page 2/2)

Q. No. 2 (vii) (Page 1/2)

$$\int_0^3 \frac{x^3+9x+3}{x^2+9}\,dx$$

$$= \int_0^3 \frac{x(x^2+9)+3}{x^2+9}\,dx$$

$$= \int_0^3 \frac{x\cancel{(x^2+9)}}{\cancel{x^2+9}}\,dx + \frac{3}{x^2+9}\,dx$$

$$= \int_0^3 x\,dx + \int_0^3 \frac{3}{x^2+9}\,dx$$

$$= \int_0^3 x\,dx + 3\int_0^3 \frac{1}{(x)^2+(3)^2}$$

$$= \left[\frac{x^2}{2}\right]_0^3 + 3\left[\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)\right]_0^3 \quad \because \frac{1}{x^2+a^2} = \tan^{-1}\left(\frac{x}{a}\right)$$

$$= \frac{1}{2}\left[x^2\right]_0^3 + \cancel{3}\cdot\frac{1}{\cancel{3}}\left[\tan^{-1}\left(\frac{x}{3}\right)\right]_0^3$$

$$= \frac{1}{2}\left[3^2-0^2\right] + \left(\tan^{-1}\left(\frac{3}{3}\right) - \tan^{-1}\left(\frac{0}{3}\right)\right)$$

$$= \frac{1}{2}(9) + \left(\frac{\pi}{4} - 0\right)$$

$$= \frac{9}{2} + \frac{\pi}{4}$$

$$= \frac{18+\pi}{4}$$

So, $\int_0^3 \frac{x^3+9x+3}{x^2+9} = \frac{18+\pi}{4} \approx 5.2853$

Q. No. 2 (vii) (Page 2/2)

Q. No. 2 (viii) (Page 1/2)

$$\frac{dy}{dx} + \frac{4xy}{4y+2} = x$$

Taking L.C.M on L.H.S;

$$\frac{(4y+2)\frac{dy}{dx} + 4xy}{(4y+2)} = x$$

$$(4y+2)\frac{dy}{dx} + 4xy = x(4y+2)$$

$$(4y+2)\frac{dy}{dx} + 4xy = 4xy + 2x$$

$$(4y+2)\frac{dy}{dx} = \cancel{4xy} + 2x - \cancel{4xy}$$

$$(4y+2)\frac{dy}{dx} = 2x$$

$$(4y+2)\,dy = (2x)\,dx$$

Integrating both sides;

$$\int (4y+2)\,dy = \int 2x\,dx$$

$$\frac{\cancel{4}^{2}y^2}{\cancel{2}_1} + 2y = \cancel{2}\cdot\frac{x^2}{\cancel{2}} + c$$

$$2y^2 + 2y = x^2 + c$$

$$2(y^2+y) = x^2 + c$$

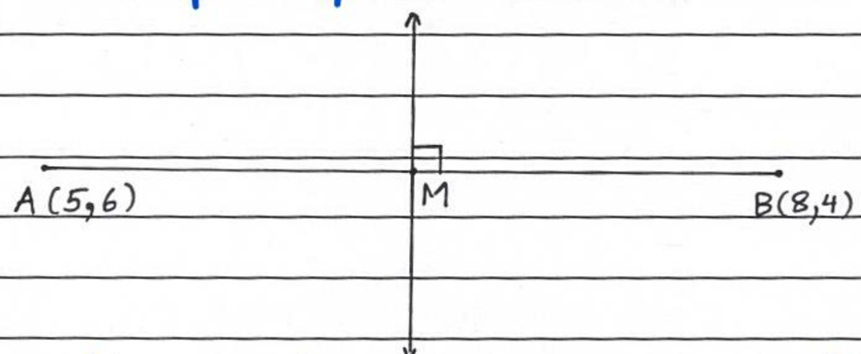
$$y^2 + y = x^2 + \frac{c}{2}$$

$$\boxed{y^2 + y = x^2 + c_1} \qquad \because c_1 = \frac{c}{2}$$

Q. No. 2 (viii) (Page 2/2)

Q. No. 2 (ix) (Page 1/2)

Equation of Perpendicular Bisector



Let $A(5,6) = (x_1, y_1)$; $B(8,4) = (x_2, y_2)$

The perpendicular bisector divides the lines in two parts and is right angle to it.

The slope of line is;

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_1 = \frac{4-6}{8-5}$$

$$m_1 = \frac{-2}{3}$$

The midpoint of the two lines is;

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$M = \left(\frac{5+8}{2}, \frac{6+4}{2}\right)$$

$$M = \left(\frac{13}{2}, \frac{10}{2}\right)$$

$$M = \left(\frac{13}{2}, 5\right)$$

Q. No. 2 (ix) (Page 2/2)

$$m_2 = \frac{-1}{m_1}$$

$$m_2 = \frac{-1}{-2/3}$$

$$m_2 = \frac{3}{2}$$

Now, equation of line perpendicular to the given line and bisecting it is;

$$y - y_1 = m(x - x_1)$$

Here, it passes through $M\left(\frac{13}{2}, 5\right)$ so;

$$y - 5 = \frac{3}{2}\left(x - \frac{13}{2}\right)$$

$$2(y-5) = 3\left(x - \frac{13}{2}\right)$$

$$2y - 10 = 3x - \frac{39}{2}$$

$$3x - 2y - \frac{39}{2} + 10 = 0$$

$$3x - 2y - \frac{19}{2} = 0$$

Xing eq by '2';

$$\boxed{6x - 4y - 19 = 0}$$

This is the required equation of line.

Q. No. 2 (x) (Page 1/2)

VALUE OF 'k'

The given lines are;

$$2x - 2y + 2 = 0 \quad -(1)$$
$$3x - 5y - 1 = 0 \quad -(2)$$
$$2x + ky + 8 = 0 \quad -(3)$$

The point at which the three lines meet can also be referred as the point of intersection of two of them;

So, the point of intersection of (1) & (2) will be;

Multiplying eq (1) by 3 and eq (2) by 2 and subtracting them;

$$6x - 6y + 6 = 0$$
$$- \quad \overset{-}{+}6x \overset{+}{-}10y \overset{+}{-}2 = 0$$
$$4y + 8 = 0$$
$$4y = -8$$
$$\boxed{y = -2} \quad -(4)$$

Put $y = -2$ in eq (1);

$$(1) \Rightarrow \quad 2x - 2(-2) + 2 = 0$$
$$2x + 4 + 2 = 0$$
$$2x + 6 = 0$$
$$2x = -6$$
$$\boxed{x = -3}$$

The point of intersection is $P(-3, -2)$.

Q. No. 2 (x) (Page 2/2)

Putting $x = -3$, $y = -2$ in eq (3);

$$(3) \Rightarrow \quad 2(-3) + k(-2) + 8 = 0$$
$$-6 - 2k + 8 = 0$$
$$-2k + 2 = 0$$
$$-2k = -2$$
$$\boxed{k = 1}$$

So, the value of 'k' is 1.

Q. No. 2 (xi) (Page 1/2)

Q. No. 2 (xi) (Page 2/2)

Q. No. 2 (xii) (Page 1/2)

Q. No. 2 (xii) (Page 2/2)

Q. No. 2 (xiii) (Page 1/2)

EQUATION OF PARABOLA

Given; Focus; $(3, 2)$

Directix; $2x-y+5$

In a parabola, the distance of point on a parabola to the focus and to the directix is equal.

So, $|\bar{PF}| = |\bar{PM}|$ —①

Let we take a point $P(x,y)$ on the parabola.

Then eq ① is;

$$\sqrt{(x-3)^2+(y-2)^2} = \frac{|2x-y+5|}{\sqrt{2^2+(-1)^2}}$$

$$\sqrt{x^2+9-6x+y^2+4-4y} = \frac{2x-y+5}{\sqrt{5}}$$

Squaring both sides of equation;

$$x^2+y^2-6x-4y+13 = \frac{(2x-y+5)^2}{(\sqrt{5})^2}$$

$(2x)^2+(-y)^2+(5)^2+2(2x)(-y)+2(-y)(5)$
$+2(2x)(5)$

$$x^2+y^2-6x-4y+13 = \frac{4x^2+y^2+25-4xy-10y+20x}{5}$$

$$5(x^2+y^2-6x-4y+13) = 4x^2+y^2+25-4xy-10y+20x$$

$$5x^2+5y^2-30x-20y+65 = 4x^2+y^2+25-4xy-10y+20x$$

$$x^2+4y^2-50x-10y+4xy+40 = 0$$

Q. No. 2 (xiii) (Page 2/2)

Q. No. 2 (xiv) (Page 1/2)

Q. No. 2 (xiv) (Page 2/2)

Q. No. 2 (xv) (Page 1/2)

FINDING 'α'

Given; Let $\underline{u} = 3\bar{i} + \alpha\bar{j} + 4\bar{k}$

$\underline{v} = 4\bar{i} + 5\bar{j} + \alpha\bar{k}$

If these vectors are perpendicular, their dot product must be zero;

$$\underline{u} \cdot \underline{v} = 0$$

$$(3\underline{i} + \alpha\underline{j} + 4\underline{k})(4\underline{i} + 5\underline{j} + \alpha\underline{k}) = 0$$

$$(3)(4) + (\alpha)(5) + (4)(\alpha) = 0$$

$$12 + 5\alpha + 4\alpha = 0$$

$$12 + 9\alpha = 0$$

$$9\alpha = -12$$

$$\alpha = -\frac{12}{9}$$

$$\boxed{\alpha = -\frac{4}{3}}$$

Hence, the value of 'α' is $-\frac{4}{3}$.

Q. No. 2 (xv) (Page 2/2)

Q. No. 2 (xvi) (Page 1/2)

Given: A (-2, 1, 4)

B (3, 2, 5)

C (-3, -5, 0)

D (5, 8, 9)

From vertices, we can calculates the sides;

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

$$= (3\underline{i} + 2\underline{j} + 5\underline{k}) - (-2\underline{i} + 1\underline{j} + 4\underline{k})$$

$$= (3-(-2))\underline{i} + (2-1)\underline{j} + (5-4)\underline{k}$$

$$\overrightarrow{AB} = 5\underline{i} + 1\underline{j} + 1\underline{k}$$

$$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$$

$$= (-3\underline{i} - 5\underline{j} + 0\underline{k}) - (-2\underline{i} + 1\underline{j} + 4\underline{k})$$

$$= (-3-(-2))\underline{i} + (-5-1)\underline{j} + (0-4)\underline{k}$$

$$\overrightarrow{AC} = -1\underline{i} - 6\underline{j} - 4\underline{k}$$

Now, we calculate the cross product of $\overrightarrow{AB}$ and $\overrightarrow{AC}$ (the adjacent sides);

$$|\overrightarrow{AB} \times \overrightarrow{AC}| = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 5 & 1 & 1 \\ -1 & -6 & -4 \end{vmatrix}$$

$$= \underline{i}\begin{vmatrix} 1 & 1 \\ -6 & -4 \end{vmatrix} - \underline{j}\begin{vmatrix} 5 & 1 \\ -1 & -4 \end{vmatrix} + \underline{k}\begin{vmatrix} 5 & 1 \\ -1 & -6 \end{vmatrix}$$

$$= \underline{i}(-4-(-6)) - \underline{j}(-20-(-1)) + \underline{k}(-30-(-1))$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = 2\underline{i} + 19\underline{j} - 29\underline{k}$$

Q. No. 2 (xvi) (Page 2/2)

$V = \frac{1}{6} |\vec{AB} \times \vec{AC}|$ — (1)

$|\vec{AB} \times \vec{AC}| = \sqrt{(2)^2 + (19)^2 + (-29)^2}$

$= \sqrt{4 + 361 + 841}$

$= \sqrt{1206}$

$|\vec{AB} \times \vec{AC}| = 3\sqrt{134}$

Putting in (1);

$V = \frac{1}{6} \times 3\sqrt{134}$

$V = \frac{\sqrt{134}}{2}$ cubic units

$\boxed{V = 5.7879 \text{ cubic units}}$

$$f(x) = \begin{cases} mx+3 & \text{if } x<3 \\ m+n & \text{if } x=3 \\ -x+9 & \text{if } x>3 \end{cases}$$

First, we calculate $f(x)$ at $x=3$. So;

$$f(x)\Big|_{x=3} = f(3) = m+n \quad (1)$$

(a)

$\lim_{x\to 3^-} f(x)$

This is the left hand limit.

$= \lim_{x\to 3} (mx+3)$

$= m(3)+3$

$= 3m+3$

So, $\lim_{x\to 3^-} f(x) = 3m+3 \quad (2)$

$\lim_{x\to 3^+} f(x)$

This is the right hand limit means that function is approaching 3 from right.

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3} (-x+9)$

$= -3+9$

$= 6$

So, $\lim_{x\to 3^+} f(x) = 6 \quad (3)$

Q. No. 3 (Page 2/4) **(b)**

As calculated before;

$$f(x)\Big|_{x=3} = f(3) = m+n$$

From eq ①, ②, ③;

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = \lim_{x\to 3} f(3)$$

$$3m+3 = 6 = m+n$$

(c)

Values of m and n

$$3m+3=6$$
$$3m = 6-3$$
$$3m = 3$$
$$\boxed{m=1}$$

$$6 = m+n$$

Put $m=1$ in eq.

$$6 = 1+n$$
$$6-1 = n$$
$$\boxed{n=5}$$

So, the value of m and n are 1 and 5 respectively.

(d)

Now, the function becomes;

$$f(x) = \begin{cases} 1x+3 & \text{if } x<3 \\ 1+5=6 & \text{if } x=3 \\ -x+9 & \text{if } x>3 \end{cases}$$

Table

x ←	0	1	2	3	4	5	→
y ←	3	4	5	6	5	4	→

Graph

The graph is drawn on Graph Page No. 1.

Q. No. 3 (Page 4/4)

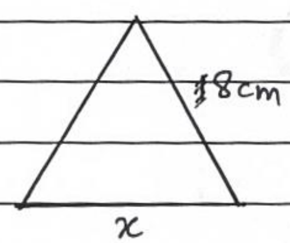
MAXIMUM AREA OF TRIANGLE

Given:

Perimeter P = 18 cm

Length of one side = 8 cm

Figure:



Solution:

Let one side of the triangle = x

Then second unknown side becomes $= 18 - x - 8$

$= 10 - x$

(a)

Here, the function is the area function squared

Area of triangle is given by Hero's formula;

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\because s = \frac{a+b+c}{2} = \frac{x + 8 + 10 - x}{2} = \frac{18}{2}$$

$$s = 9$$

Q. No. 4 (Page 2/4)

$$A^2 = 9(9-8)(9-x)(9-(10-x))$$
$$= 9(1)(9-x)(9-10+x)$$
$$= 9(9-x)(-1+x)$$
$$= 9(-9+9x+x-x^2)$$
$$= 9(-9+10x-x^2)$$
$$f(x) = A^2 = -81+90x-9x^2 \quad (1)$$

(b)

Differentiating (1) w.r.t x;

$$f'(x) = 0 + 90 - 18x$$
$$f'(x) = 90 - 18x \quad (2)$$

Differentiating eq (2) w.r.t x;

$$f''(x) = -18 \quad (3)$$

(c)

For extreme values, put $f'(x) = 0$;

$$f'(x) = 0$$
$$90 - 18x = 0$$
$$90 = 18x$$
$$\frac{90}{18} = x$$
$$\boxed{x = 5}$$

Put $x = 5$ in (2);

③ $\Rightarrow f''(x)\Big|_{x=5} = -18 < 0$

As $f''(x) < 0$ so, it is a relative maxima.
Hence, the area of triangle is maximum.

(d)

Sides of Triangle

So, the side of triangle are;

Side 1 = 8 cm

Side 2 = x = 5cm

Side 3 = $10 - x = 10 - 5 = 5$cm.

For triangle with sides 8cm, 5 cm, 5cm, it will give maximum area.

Q. No. 4 (Page 4/4)

Q. No. 5 (Page 1/4)

$$\int \frac{2x^2+5x+3}{(x-2)^2(x^2+x+1)}\,dx$$

Let $\frac{2x^2+5x+3}{(x-2)^2(x^2+x+1)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{Cx+D}{x^2+x+1}$ — (1)

Multiplying equation by $(x-2)^2(x^2+x+1)$;

$2x^2+5x+3 = A(x-2)(x^2+x+1) + B(x^2+x+1) + Cx+D(x-2)^2$ — (2)

$2x^2+5x+3 = A(x^3+x^2+x-2x^2-2x-2) + B(x^2+x+1) +$
$Cx+D(x^2-4x+4)$

$2x^2+5x+3 = A(x^3-x^2-x-2) + Bx^2+Bx+B$
$+ Cx^3-4Cx^2+4Cx+Dx^2-4Dx+4D$

$2x^2+5x+3 = Ax^3-Ax^2-Ax-2A+Bx^2+Bx+B+Cx^3-4Cx^2+4Cx$
$+Dx^2-4Dx+4D$ — (3)

Put $x-2=0$ or $x=2$ in eq (2);

$2(2)^2+5(2)+3 = A(0)(x^2+x+1) + B(2^2+2+1) + Cx+D(0)$

$21 = 7B$

$\frac{21}{7} = B$

$\boxed{B=3}$

Comparing coefficients;

x^3; $0 = A+C$ — (3) $\Rightarrow$ $A=-C$ — (7)

x^2; $2 = -A+B-4C+D$ — (4)

Q. No. 5 (Page 2/4)

Put $A=-C$ and $B=3$ in eq (4);

$$2 = -(-C) + 3 - 4C + D$$
$$2 = C + 3 - 4C + D$$
$$2 - 3 = C - 4C + D$$
$$-1 = -3C + D$$
$$1 = 3C - D \quad (8)$$

Put $A=-C$ and $B=C$ in eq (5);

$$5 = -(-C) + 3 + 4C - 4D$$
$$5 = C + 3 + 4C - 4D$$
$$5 - 3 = C + 4C - 4D$$
$$2 = 5C - 4D \quad (9)$$

$4\cancel{3} \times (8) - (9)$

$$5 = 15C - \cancel{4D}$$
$$- \quad \overset{-}{\oplus}2 = \overset{-}{\oplus}5C \ \overset{+}{\ominus} \cancel{4D}$$
$$3 = 12C$$
$$C = \frac{3}{12}$$
$$C = \frac{1}{4}$$

Put $C = \frac{1}{4}$ in (8);

$$1 = 3\left(\frac{1}{4}\right) - D$$

Q. No. 5 (Page 3/4)

Put $C = \frac{1}{4}$ in ⑦;

$$A = -\frac{1}{4}$$

Now, eq ① becomes;

$$\frac{2x^2+5x+3}{(x-2)^2(x^2+x+1)} = \frac{-1}{4(x-2)} + \frac{3}{(x-2)^2} + \frac{1x-1}{4(x^2+x+1)}$$

Integrating both sides;

$$\int \frac{2x^2+5x+3}{(x-2)^2(x^2+x+1)}\,dx = \int \frac{-1}{4(x-2)}\,dx + \int \frac{3}{(x-2)^2}\,dx + \int \frac{x-1}{4(x^2+x+1)}\,dx$$

$$= \frac{-1}{4}\int \frac{1}{(x-2)}\,dx + 3\int (x-2)^{-2}\,dx + \frac{1}{4}\int \frac{x+1-1-1}{x^2+x+1}\,dx$$

$$= \frac{-1}{4}\ln|x-2| + 3\cdot\frac{(x-2)^{-2+1}}{-2+1} + \frac{1}{4}\int \frac{x+1}{x^2+x+1} + \int \frac{-2}{x^2+x+1}$$

$$= \frac{-1}{4}\ln|x-2| - 3(x-2)^{-1} \cancel{4} - 2\int \frac{1}{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} + \frac{1}{8}\int \frac{2x+1}{x^2+x+1}$$

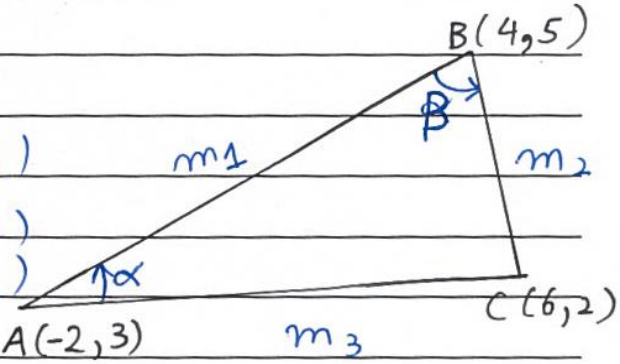
$$= \frac{-1}{4}\ln|x-2| - \frac{3}{x-2} - 2\cdot\frac{1}{\frac{\sqrt{3}}{2}}\tan^{-1}\left(\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + \frac{1}{8}\ln|x^2+x+1|$$

$$= -\frac{1}{4}\ln|x-2| - \frac{3}{x-2} - \frac{4}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + \frac{1}{8}\ln|x^2+x+1| + c$$

Q. No. 5 (Page 4/4)

Q. No. 6 (Page 1/4)

Given:



$A(-2,3) = (x_1, y_1)$
$B(4,5) = (x_2, y_1)$
$C(6,2) = (x_3, y_3)$

(a)

The formula for slope is given by;

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{Slope of } \bar{AB} = m_1 = \frac{5-3}{4-(-2)} = \frac{2}{6} = \frac{1}{3}$$

$$\text{Slope of } \bar{BC} = m_2 = \frac{2-5}{6-4} = \frac{-3}{2}$$

$$\text{Slope of } \bar{AC} = m_3 = \frac{2-3}{6-(-2)} = \frac{-1}{8}$$

(b)

Let the angle between $\bar{AB}$ and $\bar{BC}$ is β and angle between $\bar{AB}$ and $\bar{AC}$ is α.

The angle between $\bar{AB}$ and $\bar{BC}$ is;

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$\tan\beta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

Q. No. 6 (Page 2/4)

$$\tan\beta = \frac{\frac{-11}{6}}{\frac{1}{2}}$$

$$\tan\beta = \frac{-11}{3}$$

$$\beta = \tan^{-1}\left(\frac{-11}{3}\right)$$

$$\beta = 180^\circ - \tan^{-1}\left(\frac{11}{3}\right)$$

$$\beta = 180^\circ - 74.74^\circ$$

$$\boxed{\beta = 105.25^\circ}$$

$$\tan\alpha = \frac{m_1 - m_3}{1 + (m_1)(m_3)}$$

$$\tan\alpha = \frac{\frac{1}{3} - \left(\frac{-1}{8}\right)}{1 + \left(\frac{1}{3}\right)\left(\frac{-1}{8}\right)}$$

$$\tan\alpha = \frac{\frac{11}{24}}{\frac{23}{24}}$$

$$\tan\alpha = \frac{11}{23}$$

$$\alpha = \tan^{-1}\left(\frac{11}{23}\right)$$

$$\boxed{\alpha = 25.559^\circ}$$

Q. No. 6 (Page 3/4) **(c)**

Equation of side $\bar{AB}$

Side $\bar{AB}$ has slope $\frac{1}{3}$ and passes through $A(-2,3)$

$$y - y_1 = m(x - x_1)$$
$$y - 3 = \frac{1}{3}(x - (-2))$$
$$y - 3 = \frac{1}{3}(x + 2)$$
$$3(y - 3) = x + 2$$
$$3y - 9 = x + 2$$
$$x - 3y + 2 + 9 = 0$$
$$\boxed{x - 3y + 11 = 0}$$

Equation of side $\bar{BC}$

Side $\bar{BC}$ has slope $\frac{-3}{2}$ and passes through $B(4,5)$

$$y - 5 = \frac{-3}{2}(x - 4)$$
$$2y - 10 = -3x + 12$$
$$3x + 2y - 10 - 12 = 0$$
$$\boxed{3x + 2y - 22 = 0}$$

(d)

The area of triangle is;

$$\begin{vmatrix} x_1 & y_1 & 1 \end{vmatrix}$$

Q. No. 6 (Page 4/4) $A = \frac{1}{2}\begin{vmatrix} -2 & 3 & 1 \\ 4 & 5 & 1 \\ 6 & 2 & 1 \end{vmatrix}$

$= \frac{1}{2}\left[-2\begin{vmatrix} 5 & 1 \\ 2 & 1 \end{vmatrix} - 3\begin{vmatrix} 4 & 1 \\ 6 & 1 \end{vmatrix} + 1\begin{vmatrix} 4 & 5 \\ 6 & 2 \end{vmatrix}\right]$

$= \frac{1}{2}\left[-2(5-2) - 3(4-6) + 1(8-30)\right]$

$= \frac{1}{2}\left[-2(3) - 3(-2) + 1(-22)\right]$

$= \frac{1}{2}(-22)$

$\boxed{\text{Area} = 11 \text{ square units}}$ $\because$ Area is always +ve.

To check if colinear points;

$= \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$= \begin{vmatrix} -2 & 3 & 1 \\ 4 & 5 & 1 \\ 6 & 2 & 1 \end{vmatrix}$

$= -2\begin{vmatrix} 5 & 1 \\ 2 & 1 \end{vmatrix} - 3\begin{vmatrix} 4 & 1 \\ 6 & 1 \end{vmatrix} + 1\begin{vmatrix} 4 & 5 \\ 6 & 2 \end{vmatrix}$

$= -2(5-2) - 3(4-6) + 1(8-30)$

$= -2(3) - 3(-2) + 1(-22)$

$= 22$

$\neq 0$

As it is not equal to zero so points are not coolinear.

Q. No. 7 (Page 1/4)

Q. No. 7 (Page 2/4)

Q. No. 7 (Page 3/4)

Q. No. 7 (Page 4/4)

Q. No. 8 (Page 1/4)

The relevant question should be answered only in the allotted space and inside the outer mark

Q. No. 8 (Page 2/4)

Q. No. 8 (Page 3/4)

Q. No. 8 (Page 4/4)

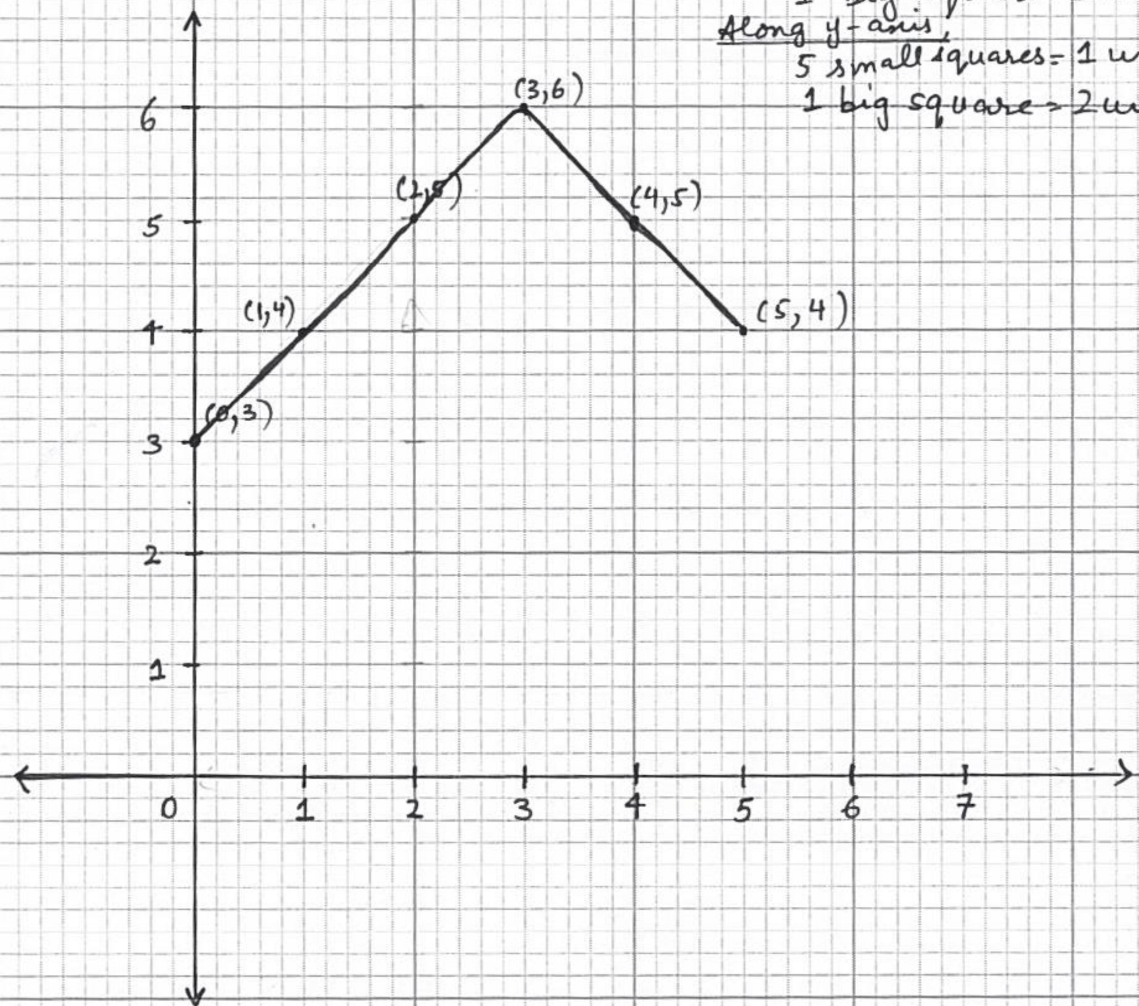
گراف پیپر: متعلقہ سوال کا سیریل نمبر ضرور درج کریں۔

Q3 (d).

SCALE:

Along x-axis;
5 small squares = 1 unit
1 big square = 2 units

Along y-axis,
5 small squares = 1 unit
1 big square = 2 units



Graph Page No. 2