(Section B)

Q. No. 2 (i) (Page 1/2)

$f(x) = \sqrt{x^3+4}$

Finding $f^{-1}(x)$

let $y = f(x)$ then $x = f^{-1}(y)$

so

$y = \sqrt{x^3+4}$

cubing both sides

$y^3 = x^3+4$

$y^3 - 4 = x^3$

take cube root on both sides.

$x = \sqrt[3]{y^3-4}$

as $x = f^{-1}(y)$ so

$f^{-1}(y) = \sqrt[3]{y^3-4}$

so

$f^{-1}(x)$ will be obtained by replacing y by x

$\boxed{f^{-1}(x) = \sqrt[3]{x^3-4}}$

Now $f(f^{-1}(x))$: $\Rightarrow$

$f(x) = \sqrt[3]{x^3+4}$

$f(f^{-1}(x)) = \sqrt[3]{(f^{-1}x)^3+4}$

$= \sqrt[3]{\left(\sqrt[3]{x^3-4}\right)^3+4}$

Q. No. 2 (i) (Page 2/2)

therefore

$$f(f^{-1}(x)) = x$$

similary $f^{-1}(f(x)) \Rightarrow$

$$f^{-1}(x) = \sqrt[3]{x^3-4}$$

$$f^{-1}(f(x)) = \sqrt[3]{(f(x))^3-4}$$

$$= \sqrt[3]{x^3+4-4} = \sqrt[3]{x^3}$$

$$= x$$

so

$$f(f^{-1}(x)) = f^{-1}(f(x)) = x.$$

$\longleftrightarrow$

Q. No. 2 (ii) (Page 1/2)

$$\lim_{x\to 0} \frac{\operatorname{cosec} x - \cot x}{x}$$

$$\lim_{x\to 0} \left(\frac{\frac{1}{\sin x} - \frac{\cos x}{\sin x}}{x} \right)$$

$$\Rightarrow \lim_{x\to 0} \left(\frac{1 - \cos x}{x.\sin x} \right)$$

$$\because \sin^2 x = \frac{1 - \cos x}{2}.$$

$$\Rightarrow \lim_{x\to 0} \left(\frac{2\sin^2 x}{x.\sin x} \right)$$

$$\Rightarrow \lim_{x\to 0} \left(\frac{2\sin x}{x} \right)$$

$$\Rightarrow 2 \lim_{x\to 0} \left(\frac{\sin x}{x} \right) \qquad \because \lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1.$$

$$\Rightarrow 2(1)$$

$$\boxed{\Rightarrow 2}$$

Q. No. 2 (ii) (Page 2/2)

Q. No. 2 (iii) (Page 1/2)

$y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \cdots}}}$

To prove => $(2y-1)\frac{dy}{dx} = \cos x$.

Sol

$y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \cdots}}}$

Squaring both sides

$y^2 = \sin x + \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \cdots}}}$

$y^2 = \sin x + y$ (given y is equal to $\sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x \cdots}}}$)

$y^2 = \sin x + y$ —(1)

differentiate (1) with respect to x

$\frac{d}{dx} y^2 = \frac{d}{dx} \sin x + \frac{d}{dx} y$

$2y\frac{dy}{dx} = \cos x + \frac{dy}{dx}$

$2y\frac{dy}{dx} - \frac{dy}{dx} = \cos x$

$(2y-1)\frac{dy}{dx} = \cos x$

hence proved

Q. No. 2 (iii) (Page 2/2)

Q. No. 2 (iv) (Page 1/2)

$$\sin(x+h) = \sin x + h\cos x - \frac{h^2}{2!}\sin x - \frac{h^3}{3!}\cos x + \cdots$$

Taylor series is as

$$f(x+h) = f(x) + f'(x)h + \frac{f''(x)h^2}{2!} + \frac{f'''(x)h^3}{3!} + \cdots$$

comparing both

$$f(x+h) = \sin(x+h)$$

$$\Rightarrow \quad f(x) = \sin x.$$

$$\Rightarrow f(x) = \sin x$$

$$f'(x) = \frac{d}{dx}\sin x = \cos x$$

$$f''(x) = \frac{d}{dx}\cos x = -\sin x$$

$$f'''(x) = \frac{d}{dx} - \sin x = -\cos x.$$

put values in taylor series.

$$\sin(x+h) = \sin x + \cos x\, h - \frac{\sin x\, h^2}{2!} - \frac{\cos x\, h^3}{3!} + \cdots$$

hence proved

Q. No. 2 (iv) (Page 2/2)

Q. No. 2 (v) (Page 1/2)

$$y = \sin^{-1}\frac{x}{a}$$

To show =>

$$y_2 = x(a^2 - x^2)^{-3/2}$$

Sol.

$$y = \sin^{-1}\frac{x}{a}$$

differentiate w.r.t x

$$y_1 = \frac{d}{dx}\sin^{-1}\frac{x}{a}.$$

$$\because \frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}\frac{d}{dx}(x)$$

$$y_1 = \frac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}}\frac{d}{dx}\left(\frac{x}{a}\right)$$

$$y_1 = \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\left(\frac{1}{a}\right)$$

$$y_1 = \frac{1}{\sqrt{\frac{a^2-x^2}{a^2}}}\frac{1}{a}$$

$$y_1 = \frac{1}{\frac{\sqrt{a^2-x^2}}{a}} \times \frac{1}{a}$$

$$y_1 = \frac{\cancel{a}}{\sqrt{a^2-x^2}} \times \frac{1}{\cancel{a}} = \frac{1}{\sqrt{a^2-x^2}} = (a^2-x^2)^{-1/2}.$$

again differentiate w.r.t x.

$$y_2 = \frac{d}{dx}(a^2 - x^2)^{-1/2}$$

Q. No. 2 (v) (Page 2/2)

$\because \frac{d}{dx} x^n = nx^{n-1}$

$$y_2 = -\frac{1}{2}(a^2-x^2)^{-1/2-1}\frac{d}{dx}(a^2-x^2)$$

$$y_2 = -\frac{1}{2}(a^2-x^2)^{\frac{-3}{2}}(-2x)$$

$$\cancel{2}y_2 = -(a^2-x^2)^{-3/2}(-2x)$$

$$\cancel{2}y_2 = \cancel{2}x(a^2-x^2)^{-3/2}$$

$$\boxed{y_2 = x(a^2-x^2)^{-3/2}.}$$

hence proved

Q. No. 2 (vi) (Page 1/2)

$$\int \frac{dx}{3x(\ln 3x)^4}$$

let $\ln 3x = t$

$\ln 3x = t$

apply differential.

$\frac{d}{dx} \ln x = \frac{1}{x}$

$\frac{3}{3x} dx = dt$

$\frac{1}{3x} dx = \frac{1}{3} dt$

put in original eqaution

$$\int \frac{dt}{3(t)^4}$$

$\Rightarrow \frac{1}{3} \int \frac{dt}{t^4}$

$\Rightarrow \frac{1}{3} \frac{t^{-4+1}}{-4+1} + c$

$\Rightarrow \frac{1}{3} \frac{t^{-3}}{-3} + c$

$\Rightarrow -\frac{1}{9} t^{-3} + c$

$\because t = \ln 3x$.

Q. No. 2 (vi) (Page 2/2)

Q. No. 2 (vii) (Page 1/2)

$$\frac{dy}{dx} + \frac{4xy}{4y+2} = x$$

=) separating variables.

$$\frac{dy}{dx} = x - \frac{4xy}{4y+2}.$$

$$\frac{dy}{dx} = \frac{x(4y+2) - 4xy}{(4y+2)}$$

$$(4y+2)\frac{dy}{dx} = \cancel{4xy} + 2x - \cancel{4xy}$$

$$(4y+2)\,dy = 2x\,dx$$

=) apply integral on both sides

$$\int 4y+2\,dy = \int 2x\,dx.$$

$$\frac{4y^2}{2} + 2y + c_1 = \frac{2x^2}{2} + c_2.$$

$$2y^2 + 2y = x^2 + c, \quad \because c = c_2 - c_1.$$

$$2y^2 + 2y = x^2 + c$$

$$\boxed{2y(y+1) = x^2 + c}$$

Q. No. 2 (vii) (Page 2/2)

$$\int_0^3 \frac{x^3+9x+3}{x^2+9}\,dx$$

$\frac{x^3+9x+3}{x^2+9}$ can be written as $x+\frac{3}{x^2+9}$.

So

$$\Rightarrow \int_0^3 x+\frac{3}{x^2+9}\,dx.$$

$$\int_0^3 x\,dx+3\int_0^3\frac{1}{x^2+3^2}\,dx$$

$$\frac{x^2}{2}\Big|_0^3+\cancel{3}\left(\frac{1}{\cancel{3}}\tan^{-1}\frac{x}{3}\Big|_0^3\right)$$

$$\left(\frac{9}{2}-0\right)+\tan^{-1}\frac{3}{3}-\tan^{-1}\frac{0}{3}.$$

$$\frac{9}{2}+\tan^{-1}1-\tan^{-1}0$$

$$\frac{9}{2}+\frac{\pi}{4}-0$$

$$\boxed{\frac{9}{2}+\frac{\pi}{4}}$$

Q. No. 2 (viii) (Page 1/2)

$$\frac{dy}{dx} + \frac{4xy}{4y+2} = x.$$

separating variables

$$\frac{dy}{dx} = x - \frac{4xy}{4y+2}$$

$$\frac{dy}{dx} = \frac{x(4y+2) - 4xy}{4y+2}$$

$$\frac{dy}{dx} = \frac{4xy + 2x - 4xy}{4y+2}$$

$$\frac{dy}{dx} = \frac{\cancel{2}x}{\cancel{2}(2y+1)}$$

$$\frac{dy}{dx} = \frac{x}{2y+1}$$

$$(2y+1)\,dy = x\,dx$$

$\Rightarrow$ apply integral on bothsides

$$\int (2y+1)\,dy = \int x\,dx$$

$$c_1 + \frac{2y^2}{2} + y = \frac{x^2}{2} + c_2$$

$$y^2 + y = \frac{x^2}{2} + c_3 \qquad \because c_3 = c_2 - c_1$$

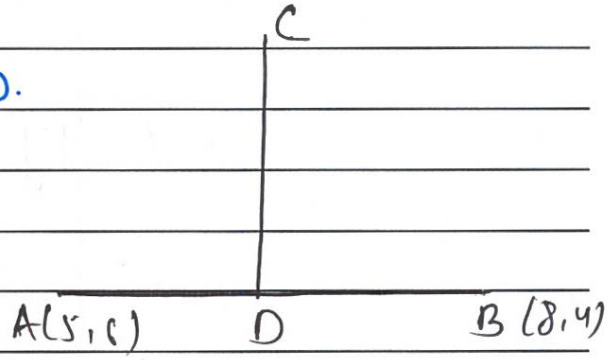
$$\boxed{2y^2 + 2y = x^2 + c} \qquad c = 2c_3.$$

Q. No. 2 (viii) (Page 2/2)

Q. No. 2 (ix) (Page 1/2)

$A(5,6)$ $B(8,4)$

required => equation of CD.



slope of AB $= \frac{4-6}{8-5}$

$m_1 = -\frac{2}{3}$

Now slope of CD $(m_2) = \frac{-1}{m_1}$ $\because CD \perp AB$.

$m_2 = \frac{-1}{(-2/3)}$

$m_2 = 3/2$.

D is midpoint of AB so

$(x_1, y_1) = \frac{5+8}{2}, \frac{6+4}{2}$.

$D = \frac{13}{2}, \frac{10}{2}$. $(x_1, y_1) = \left(\frac{13}{2}, \frac{10}{2}\right)$

Now by point slope formula

$y - y_1 = m_2(x - x_1)$

$y - \frac{10}{2} = \frac{3}{2}\left(x - \frac{13}{2}\right)$

$\frac{2y-10}{2} = \frac{3x}{2} - \frac{39}{4}$.

Q. No. 2 (ix) (Page 2/2)

$4y - 40 = 6x - 39.$

$4y = 6x + 1$

$\boxed{6x - 4y + 1 = 0}$

is req equation

Q. No. 2 (x) (Page 1/2)

$l_1 \Rightarrow 2x - 2y + 2 = 0$

$l_2 \Rightarrow 3x - 5y - 1 = 0$

$l_3 \Rightarrow 2x + ky + 8 = 0$

as lines are concurrent so

$$\begin{vmatrix} 2 & -2 & 2 \\ 3 & -5 & -1 \\ 2 & k & 8 \end{vmatrix} = 0$$

expand

$2(-40 + k) + 2(24 + 2) + 2(3k + 10) = 0$

$-80 + 2k + 52 + 6k + 20 = 0$

$8k - 8 = 0$

$8k = 8$

$k = 8/8$

$\boxed{k = 1}$

Q. No. 2 (x) (Page 2/2)

Q. No. 2 (xi) (Page 1/2)

$5x + 7y \leq 35$ —(1)

$-x + 3y \leq 3$ —(2)

$x \geq 0, y \geq 0$

associated eqs	x-intercept	y-intercept
$5x + 7y = 35$	$(7, 0)$	$(0, 7)$
$-x + 3y = 3$	$(-3, 0)$	$(0, 1)$

testing point ~~x~~ $(x, y) = (0, 0)$ on

eq (1) $\Rightarrow$

$0 \leq 35$ (True)

shading towards origin.

eq (2) $\Rightarrow$

$0 \leq 3$ (True) shading towards origin.

graph is on graph page (1) in end.

Q. No. 2 (xi) (Page 2/2)

Q. No. 2 (xii) (Page 1/2)

$A(2,3)$

$B(0,2)$

$3x+2y-3=0$

let circle be

$x^2+y^2+2gx+2fy+c=0$ — (1)

as points A & B lie on (1) so

at point A (2,3)

(1) ⇒

$4+9+2g(2)+2f(3)+c=0$

$13+4g+6f+c=0$ — (a)

at point B(0,2)

(1) ⇒

$0+4+0+4f+c=0$

$4+4f+c=0$ — (b)

The centre lies on line $3x+2y-3=0$

so

centre ⇒ $(-g,-f)$

$3x+2y-3=0$

⇒ $3(-g)+2(-f)-3=0$

$-3g-2f-3=0$ — (c).

subtract (a) & (b)

$13+4g+6f+c=0$

$\underline{4+4f+c=0}$

(d)

Q. No. 2 (xii) (Page 2/2) add (c) & (d)

$-3g-2f-3=0$

$4g+2f+9=0$

$g+6=0$

$\boxed{g=-6}$

c => $-3g-2f-3=0$

$-3(-6)-2f-3=0$

$18-3=2f$

$15=2f$

$\boxed{f=15/2}$

eq a =>

$13+4g+6f+c=0$

$13+4(-6)+6\left(\frac{15}{2}\right)+c=0$

$13-24+45+c=0$

$34+c=0$

$\boxed{c=-34}$

put in (1)

$x^2+y^2+2(-6)x+2\left(\frac{15}{2}\right)y+(-34)=0$

$\boxed{x^2+y^2-12x+15y-34=0}$

Q. No. 2 (xiii) (Page 1/2)

$\text{Focus} = (3, 2)$

$2x - y + 5 = 0$

let $P(x, y)$ be any point on parabola.

$|PF|$ = perpendicular distance from P to directrice.

$$\sqrt{(x-3)^2 + (y-2)^2} = \frac{|2x - y + 5|}{\sqrt{5}}$$

$$(x-3)^2 + (y-2)^2 = \frac{(2x-y+5)^2}{5}$$

$$\Rightarrow 5(x^2 + 9 - 6x + y^2 + 4 - 4y) = 4x^2 + y^2 + 25 - 4xy - 10y + 20x$$

$$5x^2 + 45 - 30x + 5y^2 + 20 - 20y = 4x^2 + y^2 + 25 - 4xy - 10y + 20x$$

$$\Rightarrow x^2 + 4y^2 + 65 - 30x - 20y = 25 - 4xy - 16y + 20x$$

$$\Rightarrow \boxed{x^2 + 4y^2 + 4xy - 50x - 4y + 40 = 0}$$

Q. No. 2 (xiii) (Page 2/2)

Q. No. 2 (xiv) (Page 1/2)

$9x^2 - 4y^2 = 36$

$\frac{x^2}{4} - \frac{y^2}{9} = 1.$

given line $3x + 2y + 7 = 0$

slope $= -3/2.$

slope of tangent $= m = -3/2$

tangent is given by

$y = mx \pm \sqrt{a^2m^2 - b^2}$

$a^2 = 4 \quad b^2 = 9$

$y = -\frac{3}{2}x \pm \sqrt{4\left(\frac{9}{4}\right) - 9}$

$y = -\frac{3}{2}x \pm 0$

$y = -\frac{3}{2}x$

$\boxed{2y + 3x = 0.}$

Q. No. 2 (xiv) (Page 2/2)

Q. No. 2 (xv) (Page 1/2)

let $\vec{u} = (3, \alpha, 4)$

$\vec{v} = (4, 5, \alpha)$.

as they are perpendicular so

$\vec{u} \cdot \vec{v} = 0$

$(3, \alpha, 4) \cdot (4, 5, \alpha) = 0$

$12 + 5\alpha + 4\alpha = 0$

$9\alpha = -12$

$\alpha = -12/9$

$\boxed{\alpha = -4/3}$

Q. No. 2 (xv) (Page 2/2)

Q. No. 2 (xvi) (Page 1/2)

volume is given by

$$\cancel{V = \frac{1}{6}\begin{vmatrix} -2 & 1 & 4 \\ 3 & 2 & 5 \\ -3 & -5 & 0 \end{vmatrix}}$$

vertices are

A $(-2, 1, 4)$

B $(3, 2, 5)$

C $(-3, -5, 0)$

D $(5, 8, 9)$.

$\vec{AB} = (3, 2, 5) - (-2, 1, 4) = (5, 1, 1)$

$\vec{AC} = (-3, -5, 0) - (-2, 1, 4) = (-1, -6, -4)$

$\vec{AD} = (5, 8, 9) - (-2, 1, 4) = (7, 7, 5)$

volume is given by $\frac{1}{6}[\vec{AB}\ \vec{AC}\ \vec{AD}]$.

$$V = \frac{1}{6}\begin{vmatrix} 5 & 1 & 1 \\ -1 & -6 & -4 \\ 7 & 7 & 5 \end{vmatrix}$$

$$V = \frac{1}{6}\left[5(-30+28) - 1(-5+28) + 1(-7+42)\right]$$

$$V = \frac{1}{6}\left[-10 - 23 + 35\right]$$

Q. No. 2 (xvi) (Page 2/2)

متعلقہ سوال کا جواب صرف مختص کردہ جگہ پر اور بیرونی نشان کے اندر دیا جائے۔

(Section C)

Q. No. 3 (Page 1/4)

$$f(x) = \begin{cases} mx + 3 & \text{if } x < 3 \\ m + n & \text{if } x = 3 \\ -x + 9 & \text{if } x > 3 \end{cases}$$

a

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (mx + 3)$$

$$= 3m + 3. \quad — (a)$$

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} -x + 9$$

$$= -3 + 9$$

$$= 6. \quad — (b).$$

b

$$\lim_{x \to 3} f(x) = f(3).$$

$$\lim_{x \to 3} (-x + 9) = m + n$$

$$-3 + 9 = m + n$$

$$6 = m + n \quad — (c)$$

c

as function is continuous

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x).$$

$$3m + 3 = 6$$

$$3m = 3$$

Q. No. 3 (Page 2/4)

also

$$\lim_{x \to 3} f(x) = f(3)$$

$6 = m + n$ (from part b)

~~6 = 3 +~~

$6 = 1 + n$

$n = 6 - 1$

$\boxed{n = 5}$

d **graph.**

$y = mx + 3 = x + 3$ $\quad x < 3$

$y = m + n = 6.$ $\quad x = 3.$

$y = -x + 9$ $\quad x > 3.$

table for

$y = x + 3$ $\quad x < 3.$

x	-1	0	1	2	3
y	2	3	4	5	6

for $y = -x + 9$ $\quad x > 3.$

x	3	4	5	6
y	6	5	4	3.

$\Rightarrow$ graph is on 'graph page (2)' in end.

Q. No. 3 (Page 3/4)

Q. No. 3 (Page 4/4)

Q. No. 4 (Page 1/4)

a

let one side is 8 cm so other sides are x & y.

$8 + x + y = 18$
$x + y = 10$
$y = 10 - x$

Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$s = \frac{18}{2} = 9.$

let Δ represent area ~~sa~~

$\Delta = 9(9-8)(9-x)(9-y)$
$\Delta = 9(1)(9-x)(9-10+x)$ $\because y = 10 - x.$
$\Delta = 9(9-x)(-1+x)$
$\Delta = 9(-9+9x+x-x^2)$
$\Delta = 9(-9+10x-x^2)$
$\Delta = -81 + 90x - 9x^2$

is the required function.

b

$f(x) = -81 + 90x - 9x^2$
$f'(x) = 0 + 90 - 9(2x)$
$= 90 - 18x$

again differentiate

$f''(x) = 0 - 18$

Q. No. 4 (Page 2/4)

c

for values set $f'(x) = 0$

$$90 - 18x = 0$$
$$90 = 18x$$
$$x = 5$$

~~put~~ at $x = 5$ we have $f''(x) < 0$
so we have maximum area of triangle

d

we have sides
$8, x, y$

$x = 5$
$y = 10 - x$
$y = 5$

so sides are of 8cm, 5cm & 5cm

Q. No. 4 (Page 3/4)

Q. No. 4 (Page 4/4)

Q. No. 5 (Page 1/4)

$$\int \frac{2x^2+5x+3}{(x-2)^2(x^2+x+1)}\,dx.$$

a

$$\frac{2x^2+5x+3}{(x-2)^2(x^2+x+1)} = \frac{A}{(x-2)} + \frac{B}{(x-2)^2} + \frac{(Cx+D)}{(x^2+x+1)} \quad -(1)$$

multiply both sides by $(x-2)^2(x^2+x+1)$

$$2x^2+5x+3 = A(x-2)(x^2+x+1) + B(x^2+x+1) + (Cx+D)(x-2)^2.$$

put $x=2$

$\Rightarrow 2(4)+5(2)+3 = A(0) + B(4+2+1) + 0$

$\Rightarrow 8+10+3 = B(7)$

$\Rightarrow 21 = 7B$

$\boxed{B=3}$

Now expanding equation.

$$2x^2+5x+3 = A(x-2)(x^2+x+1) + B(x^2+x+1) + (Cx+D)(x-2)^2$$

$\Rightarrow$

$$2x^2+5x+3 = A(x^3+x^2+x-2x^2-2x-2) + B(x^2+x+1) + (Cx+D)(x^2-4x+4)$$

Q. No. 5 (Page 2/4)

comparing coefficients of variable on both side.

x^3

$$0 = A + C \quad -(a)$$

x^2

$$2 = -A + B - 4C + D \quad -(b)$$

x

$$5 = -A + B + 4C - 4D \quad -(c)$$

constants

$$3 = -2A + B + D \quad -(d)$$

put $B = 3$ in (b), (c) & (d)

(b) $\Rightarrow$ $2 = -A + 3 - 4C + D$

$$-1 = -A - 4C + D \quad -(e)$$

(c) $\Rightarrow$ $5 = -A + 3 + 4C - 4D$

$$2 = -A + 4C - 4D \quad -(f)$$

(d) $\Rightarrow$ $\cancel{3} = -2A + \cancel{3} + D$

$$D - 2A = 0 \quad (g)$$

solving equations a, e, f, g

put $C = -A$ in (e)

$\Rightarrow$ $-1 = -A - 4(-A) + D$

$-1 = -A + 4A + D$

$-1 = 3A + D.$

~~multiply~~ subtract from (g).

Q. No. 5 (Page 3/4)

$D - 2A = 0$

$\cancel{+}3A \quad \cancel{+}D = 1$

$-5A = -1$

$\boxed{A = 1/5}$

as $C = -A$

$\boxed{C = -1/5}$

$D = 2A$ from (9)

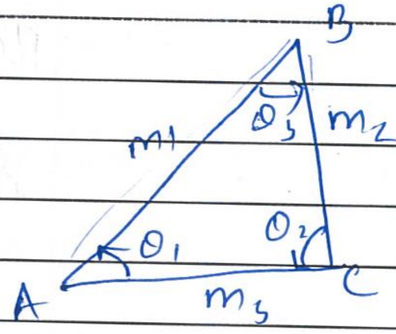
$\boxed{D = 2/5}$

put a, b, c, d in (I)

Q. No. 5 (Page 4/4)

Q. No. 6 (Page 1/4)

A $(-2,3)$
B $(4,5)$
C $(6,2)$



a

slope of AB $= m_1 = \frac{5-3}{4+2} = \frac{2}{6} = \frac{1}{3}$

slope of BC $= m_2 = \frac{2-5}{6-4} = \frac{-3}{2}$

slope of AC $= m_3 = \frac{2-3}{6+2} = \frac{-1}{8}$.

b angle between AB & BC $= \theta_3 = ?$

$$\tan\theta_3 = \frac{m_2 - m_1}{1 + m_1 m_2}$$

$$= \frac{\frac{-3}{2} - \frac{1}{3}}{1 + \left(\frac{-3}{2}\right)\left(\frac{1}{3}\right)}$$

$$\tan\theta_3 = \frac{-11/6}{1 - 1/2}$$

$$= \frac{-11/6}{1/2}$$

$$= -11/3$$

Q. No. 6 (Page 2/4)

acute angle = 74.74°

Now angle between $\overrightarrow{AB}$ & $\overrightarrow{AC}$ = θ_1 = ?

$$\tan\theta_1 = \frac{m_1 - m_3}{1 + m_1 m_3}$$

$$= \frac{\frac{1}{3} + \frac{1}{8}}{1 + \left(\frac{1}{3}\right)\left(\frac{-1}{8}\right)}$$

$$= \frac{11/24}{23/24}$$

$$= 11/23$$

$$\theta_1 = \tan^{-1} 11/23$$

$$\boxed{\theta_1 = 43.78°}$$

C

slope of $\overrightarrow{AB} = \frac{1}{3}$ at point A (-2,3)

eq is by point slope form

$$y - 3 = \frac{1}{3}(x + 2)$$

$$3y - 9 = x + 2$$

$$\boxed{x - 3y + 11 = 0}$$

Q. No. 6 (Page 3/4)

Now slope of BC = $-\frac{3}{2}$ and B (4,5)

eq =)

$$y-5 = -\frac{3}{2}(x-4)$$

$$2y-10 = -3x+12$$

$$\boxed{2y+3x-22=0}$$

d

Area =)

$$\frac{1}{2}\begin{vmatrix} -2 & 3 & 1 \\ 4 & 5 & 1 \\ 6 & 2 & 1 \end{vmatrix}$$

$$=) \frac{1}{2}\left[-2(5-2)-3(4-6)+1(8-30)\right]$$

$$=) \frac{1}{2}\left(-2(3)-3(-2)-28\right)$$

$$=) \frac{1}{2}(-6+6-28)$$

$$=) -14$$

area $\neq 0$ so points are not collinea

Q. No. 6 (Page 4/4)

Q. No. 7 (Page 1/4)

let chairs be x &
tables be y.

$x+y \leq 28$ —(1).
$480x+300y \leq 12000$ —(2).
$x \geq 0, y \geq 0$ → non negative constraints.
$p(x,y) = 200x + 150y$
we have to maximize it.

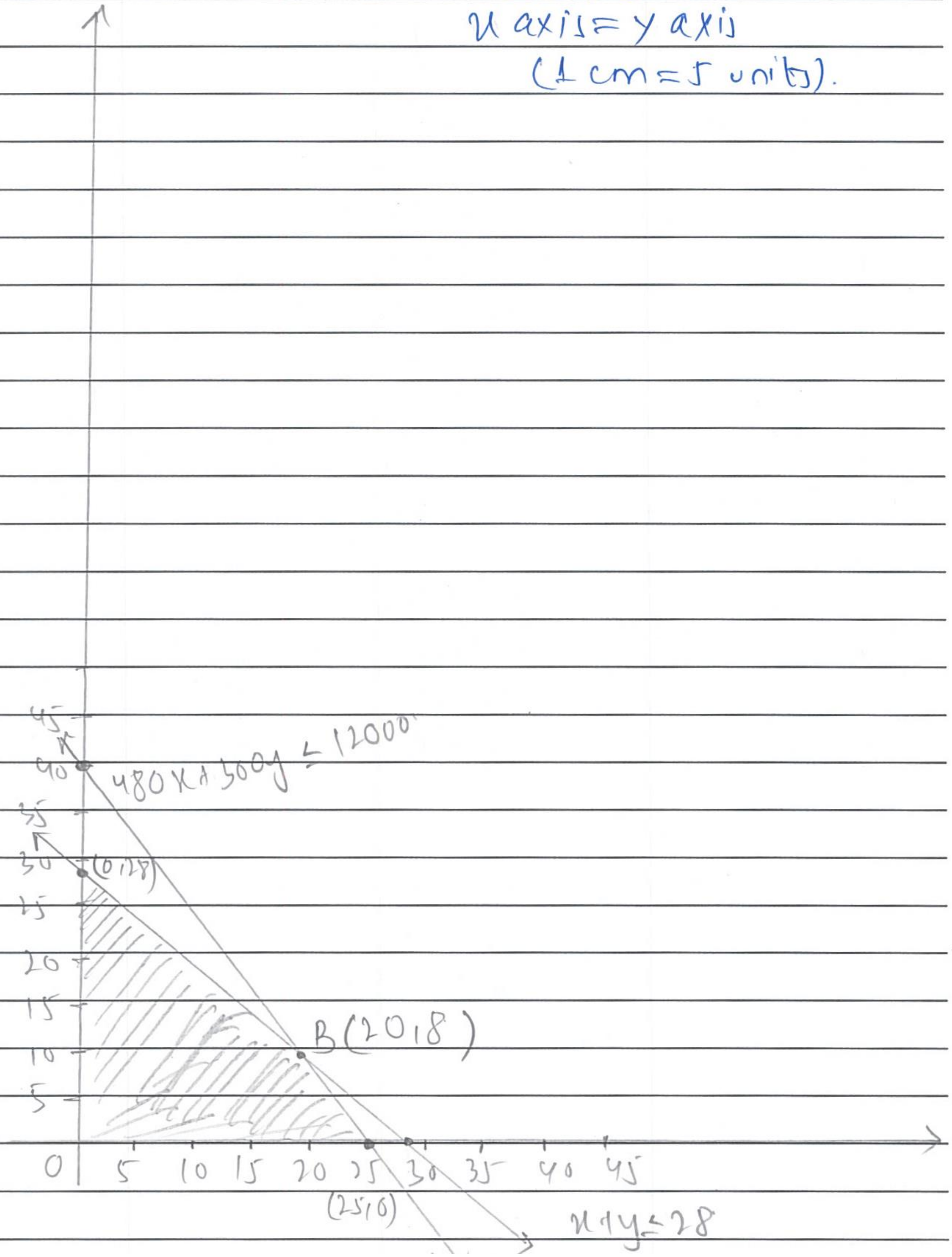
associated equation	x-intercept	y-intercept
$x+y=28$	$(28,0)$	$(0,28)$
$480x+300y=12000$	$(25,0)$	$(0,40.)$

Testing point $(x,y)=(0,0)$ on
(1) ⇒ $0 \leq 28$ ⇒ True (shading towards origin).
(2) ⇒ $0 \leq 120000$ ⇒ True (shading towards origin)

graph is on next page

Q. No. 7 (Page 2/4)

Scale
x axis = y axis
(1 cm = 5 units).



Q. No. 7 (Page 3/4)

B is intersection of

$x + y = 28$ & — (1)

$480x + 300y = 12000$

$20(24x + 15y) = 12000$

$24x + 15y = 600$

$8x + 5y = 200$ — (2)

multiply (1) by 5

$5x + 5y = 140$

subtract from (2)

$8x + 5y = 200$

$\cancel{8}5x + 5y = 140$

$3x = 60$

$x = 20$

(1) ⇒ $y = 28 - x$

$y = 8$

so

corner points	$P(x,y) = 200x + 150y$
(0, 28)	$200(0) + 150(28) = 4200$
(25, 0)	$200(25) + 150(0) = 5000$
(20, 8)	$200(20) + 150(8) = 5200$ (max)

So To maximize profit he should sell 20 chairs & 8 tables

Q. No. 7 (Page 4/4)

Q. No. 8 (Page 1/4)

$25x^2+4y^2-250x-16y+541=0$

$25x^2-250x+4y^2-16y+541=0$

$25(x^2-10x)+4(y^2-4y)+541=0$

$25(x^2-10x+25-25)+4(y^2-4y+4-4)+541=0$

$25(x^2-10x+25)-625+4(y^2-4y+4)-16+541=0$

$25(x-5)^2+4(y-2)^2=100$

$\frac{(x-5)^2}{4}+\frac{(y-2)^2}{25}=1.$ — (a)

This is ellipse.

$\frac{X^2}{4}+\frac{Y^2}{25}=1.$ —(1) $\because X=x-5$

$Y=y-2$

$b^2=4 \Rightarrow b=2$

$a^2=25 \Rightarrow a=5$

$c^2=a^2-b^2$

$c^2=21 \Rightarrow c=\sqrt{21}.$

Centre

centre (h,k) of (1) is $(0,0)$

$X=0$ $\quad Y=0$

$x-5=0$ $\quad y-2=0$

$x=5$ $\quad y=2$

Q. No. 8 (Page 2/4)

Foci

Foci of (1) are

(X, Y) $(0, \pm c)$

$X = 0$ $Y = \pm c$

$X = 0$ $Y = \pm \sqrt{21}$.

$x - 5 = 0$ $y - 2 = \pm\sqrt{21}$

$x = 5$ $y = 2 \pm \sqrt{21}$.

$\boxed{(x, y) = (5, 2 \pm \sqrt{21})}$

Eccentricity:-

$e = c/a$

$e = \frac{\sqrt{21}}{5}$

$e = 0.916 < 1.$

vertices:-

$(X, Y) = (0, \pm a)$

$X = 0$ $Y = \pm 5$

$x - 5 = 0$ $y - 2 = \pm 5$

$x = 5$ $y = 5 + 2, -5 + 2$

$y = 7, -3.$

$(x, y) = (5, 7)$ &

$(x, y) = (5, -3)$

Q. No. 8 (Page 3/4)

equation of Directrices:-

$$Y = \pm c/e^2$$

$$Y = \pm \frac{a^2}{c}. \qquad \because e = c/a$$

$$Y = \pm \frac{25}{\sqrt{21}}$$

$$y - 2 = \pm \frac{25}{\sqrt{21}}$$

$$\boxed{y = 2 \pm \frac{25}{\sqrt{21}}}$$

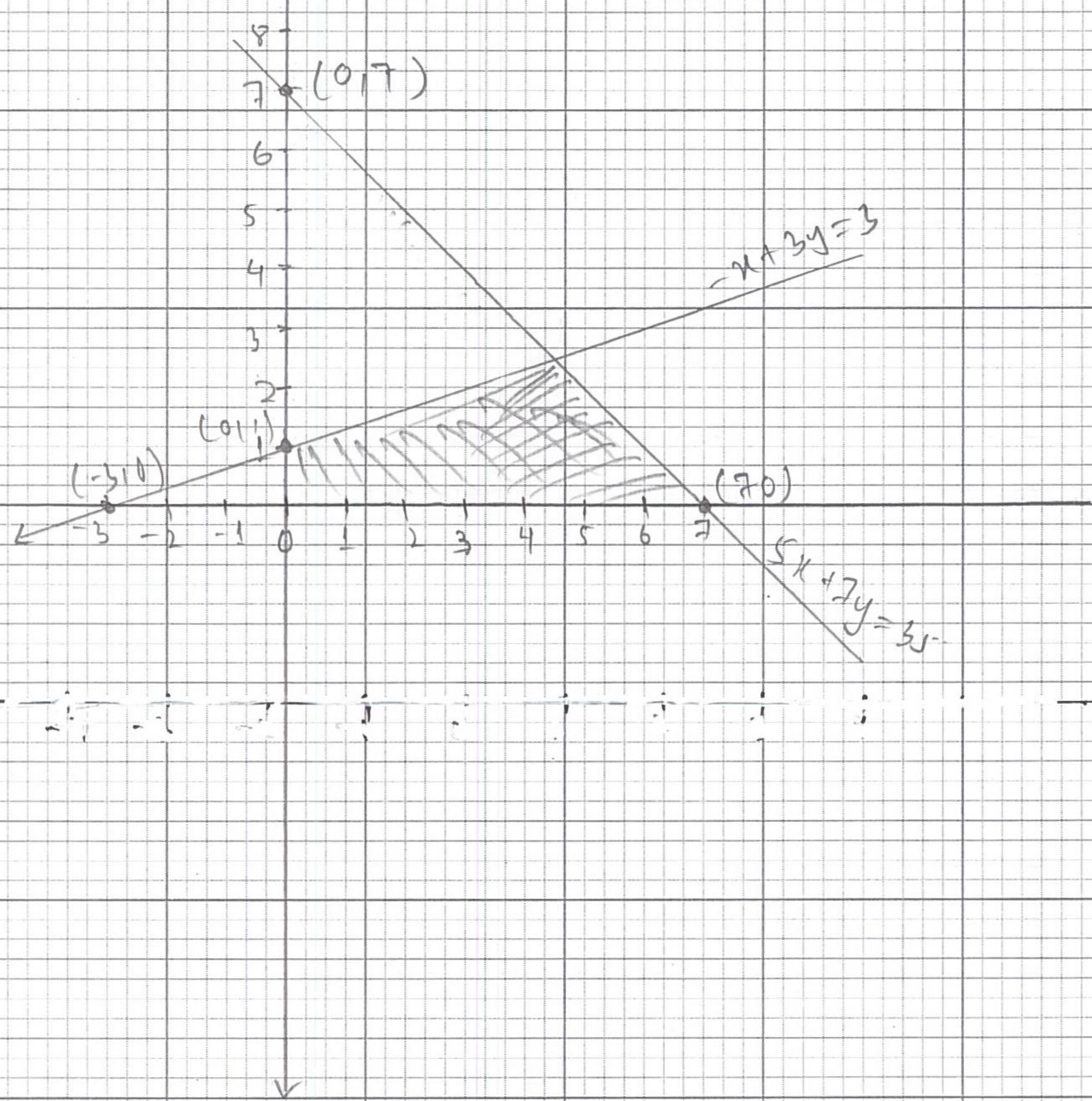
Q. No. 8 (Page 4/4)

Graph Page No. 1

Q No 02
(xi)

x-axis => 3 small boxes
(1 unit)

y-axis => 3 small boxes
(1-unit)



QN10b

d

y-axis 5 small boxes = 1 unit
x-axis 5 small boxes = 1 unit