MATHEMATICS HSSC–II

SECTION – A (Marks 20)

Time allowed: 25 Minutes

NOTE: Section–A is compulsory and comprises pages 1–2. All parts of this section are to be answered on the question paper itself. It should be completed in the first 25 minutes and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. Do not use lead pencil.

Q. 1 Circle the correct option i.e. A / B / C / D. Each part carries one mark.

(i) If domain of $f(x) = +\sqrt{4-x^2}$ is $[-2,2]$ then which of the following is the graph of $f(x)$?

    A. Semi-circle   B. Square   C. Circle   D. Hyperbola

(ii) If $f(x) = \begin{cases} 4-2x & , x<1 \\ 6x-4 & , x\geq 1 \end{cases}$ then what result comes in evaluating $\underset{x\to i}{Lim}\ f(x)$?

    A. -2 B. -1 C. 1 D. 2

(iii) if $f(x) = 2x+1$ and $g(x) = x^2 -1$ then which one of the following defines $gof(x)$?

    A. $4x^2 -4x$ B. $4x^2 +2x$ C. $(2x+1)^2 +1$ D. $(2x+1)^2 -1$

(iv) What is the value of $(1+x^2)\dfrac{d}{dx}(\tan^{-1}x - \cot^{-1}x)$?

    A. -1 B. 0 C. 1 D. 2

(v) What is the fourth derivative of $-\dfrac{1}{6}x^3 + \dfrac{1}{4}x^2 +2x+7$?

    A. 7 B. 2 C. 0 D. -1

(vi) What is the derivative of $\sin \pi$?

    A. $\cos \pi$ B. $-\cos \pi$ C. $\cos 0$ D. 0

(vii) If $f(x) = \sin^{-1}x$ then what is the value of $f'(0)$?

    A. -1 B. 0 C. 1 D. ∞

(viii) What is the evaluated value of $\int_{-1}^{1}(x^{\frac{1}{3}}+1)dx$?

    A. $\dfrac{3}{4}$ B. $\dfrac{1}{2}$ C. $\dfrac{4}{3}$ D. 2

(ix) What is the value of dy if $f(x) = x^2$ when $x=2$ and $dx = 0.001$?

    A. 0.004 B. 0.04 C. 0.001 D. 0.0401

(x) What is the solution of a differential equation $\dfrac{dy}{dx} = \sinh x$?

A. $y = -\cosh x + c$ B. $y = \cosh x + c$

C. $y = \sinh x + c$ D. $y = \cos echx + c$

(xi) What is the evaluated result of $\displaystyle\int \dfrac{\sec^2(l\,n\,x)}{x}\,dx$?

A. $\tan x + c$ B. $\sec(l\,n\,x) + c$ C. $\sec(\tfrac{1}{x}) + c$ D. $\tan(l\,n\,x) + c$

(xii) What is the x-intercept of the line $3x + 2y + 6 = 0$?

A. 2   B. -3 C. -2 D. 3

(xiii) What is the equation of a horizontal line through $(2,2)$?

A. $x = 2$ B. $y = 2$ C. $x = y$ D. $2x + 2y = 0$

(xiv) For what value of h, $(3,2)$ is the midpoint of $(2,h)$ and $(4,3h)$?

A. 3   B. 2   C. 1   D. 0

(xv) $(-1,-1)$ is a solution of which inequality given in the following?

A. $-x - 2y < 0$ B. $-4x + 3y > 0$ C. $2x - y > 10$ D. $-2x + y < -1$

(xvi) What is the length of the Latus Rectum of a parabola $12y^2 = -48x$?

A. 12   B. 4   C. -4 D. 16

(xvii) What are the coordinates of the centre of a circle $x^2 + y^2 - 8x + 12y + 21 = 0$?

A. $(-2,3)$ B. $(2,3)$ C. $(4,-6)$ D. $(0,0)$

(xviii) What is the length of the major axis of an ellipse $\dfrac{(x-1)^2}{2^2} + \dfrac{(y+1)^2}{3^2} = 1$?

A. 6   B. 4   C. 18   D. 2

(xix) For what value of "a" $2\underline{i} + a\underline{j} + 5\underline{k}$ is perpendicular to $3\underline{i} + \underline{j} + a\underline{k}$?

A. -1 B. 0   C. 1   D. 2

(xx) What are the Direction Cosines of x-axis?

A. $[0,0,0]$ B. $[1,0,0]$ C. $[0,0,1]$ D. $[0,1,0]$

For Examiner's use only:

Total Marks: 20

Marks Obtained:

MATHEMATICS HSSC–II

Time allowed: 2:35 Hours   **Total Marks Sections B and C: 80**

NOTE: Attempt any ten parts from Section 'B' and any five questions from Section 'C' on the separately provided answer book. Use supplementary answer sheet i.e. Sheet–B if required. Write your answers neatly and legibly. Graph paper will be provided on Demand.

SECTION – B (Marks 40)

Q. 2   Attempt any TEN parts. All parts carry equal marks.   **(10 x 4 = 40)**

(i)   Evaluate $\displaystyle \lim_{x \to 0} \frac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1}$; $x < 0$

(ii)   Find the extreme values of $f(x) = 1 - x^3$.

(iii)   Find $\dfrac{dy}{dx}$ if $y = x\sin^{-1}\left(\dfrac{x}{a}\right) + \sqrt{a^2 - x^2}$

(iv)   If $y = e^x . \sin x$ then show that $\dfrac{d^2 y}{dx^2} - 2\dfrac{dy}{dx} + 2y = 0$

(v)   Discuss the continuity of: $f(x) = \begin{cases} 3x - 1 & , \ x < 1 \\ 4 & , \ x = 1 \\ 2x & , \ x > 1 \end{cases}$ at $x = 1$

(vi)   Evaluate $\displaystyle \int \frac{e^x (1 + \sin x)}{(1 + \cos x)} \, dx$

(vii)   Evaluate $\displaystyle \int_1^3 \frac{x^2 - 2}{x + 1} \, dx$

(viii)   Find the area between the x-axis and the curve $y^2 = 4 - x$ in the first quadrant from $x = 0$ to $x = 3$

(ix)   Find an equation of the line through $(-4, -6)$ and perpendicular to a line having slope $-\dfrac{3}{2}$

(x)   The points $(4, -2)$, $(-2, 4)$ and $(5, 5)$ are the vertices of a triangle. Find in-centre of the triangle.

(xi)   Find the number "z" so that the triangle with vertices $A(1, -1, 0)$, $B(-2, 2, 1)$ and $C(0, 2, z)$ is a right triangle with right angle at C.

(xii)   Find equations of the tangents to the circle $x^2 + y^2 = 2$ parallel to the line $x - 2y + 1 = 0$.

(xiii)   Find focus, vertex and directrix of the parabola $x^2 - 4x - 8y + 4 = 0$.

(xiv)   Find an equation of the hyperbola with foci $(2 \pm 5\sqrt{2}, -7)$ and length of the transverse axis 10.

SECTION – C (Marks 40)

Note: Attempt any FIVE questions. All questions carry equal marks.   **(5 x 8 = 40)**

Q. 3   If θ is measured in radian, then show that $\displaystyle \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$

Q. 4   Show that $\cos(x + h) = \cos x - h\sin x - \dfrac{h^2}{\lfloor 2} \cos x + \dfrac{h^3}{\lfloor 3} \sin x + \ldots\ldots$ and evaluate $\cos 61^\circ$

Q. 5   Show that $\displaystyle \int e^{ax} \sin bx \, dx = \frac{1}{\sqrt{a^2 + b^2}} e^{ax} \sin\left(bx - \tan^{-1}\frac{b}{a}\right) + C$

Q. 6   The points $A(-1, 2)$, $B(3, -1)$ and $C(6, 3)$ are the consecutive vertices of a rhombus. Find the fourth vertex and show that the diagonals of the rhombus are perpendicular to each other.

Q. 7   Maximize $f(x, y) = x + 3y$ subject to the constraints

$2x + 5y \le 30$, $5x + 4y \le 20$, $x \ge 0$, $y \ge 0$

Q. 8   If $\underline{a} = 4\underline{i} + 3\underline{j} + \underline{k}$ and $\underline{b} = 2\underline{i} - \underline{j} + 2\underline{k}$ then find a unit vector perpendicular to both $\underline{a}$ and $\underline{b}$. Also find sine of the angle between the vectors $\underline{a}$ and $\underline{b}$.

Q. 9   Find equations of the circles of radius '2' and tangent to the line $x - y - 4 = 0$ at $(1, -3)$.