

Roll No.

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Answer Sheet No. _____

Sig. of Candidate. _____

Sig. of Invigilator. _____

29

MATHEMATICS HSSC-II

SECTION - A (Marks 20)

Time allowed: 25 Minutes

NOTE: Section-A is compulsory and comprises pages 1-2. All parts of this section are to be answered on the question paper itself. It should be completed in the first 25 minutes and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. Do not use lead pencil.

Q. 1 Circle the correct option i.e. A / B / C / D. Each part carries one mark.

(i) Domain of $f(x) = \sqrt{x^2 - 9}$ is:

- A. $R - (-3, 3)$ B. $R - [-3, 3]$ C. $[3, \infty)$ D. $(-\infty, -3]$

(ii) $\lim_{x \rightarrow 0} (x+1)^{\frac{1}{x}}$ is:

- A. 1 B. e C. ∞ D. 0

(iii) Value of $\frac{d^2}{dx^2}(-\cos x)$ at $x = \frac{\pi}{4}$ is:

- A. $\frac{1}{\sqrt{2}}$ B. $-\frac{1}{\sqrt{2}}$ C. $\frac{1}{2}$ D. $-\frac{1}{2}$

(iv) $\frac{d}{dx}(7^x) = ?$

- A. $7^x \ln x$ B. $7^x \ln 7$ C. $7^x \ln x$ D. None of these

(v) $\frac{d}{dx} \left[\sqrt{x} + \frac{1}{\sqrt{x}} \right]^2$ is:

- A. $1 - \frac{1}{x^2}$ B. $\frac{1}{2} + \frac{1}{2x^2}$ C. $\frac{1}{2} - \frac{1}{2x^2}$ D. $1 + \frac{1}{\sqrt{x}}$

(vi) $\int \left(\frac{1}{x} + \sec^2 x \right) dx = ?$

- A. $\ln x + \sec x + c$ B. $x + \tan x + c$
C. $\ln x + \tan x + c$ D. $x \tan x + c$

(vii) When $f(x) = 4x - x^2$ then $f(x)$ is increasing for the interval:

- A. $(-\infty, 2)$ B. $[-2, 2]$ C. $(2, \infty)$ D. $(-\infty, \infty)$

(viii) $\int \frac{e^{\sin^{-1} x}}{\sqrt{1-x^2}} dx = ?$

- A. $e^{\cot^{-1} x} + C$ B. $e^{\cos^{-1} x} + C$ C. $e^{\sin^{-1} x} + C$ D. $e^{\tan^{-1} x} + C$

(ix) $\int_0^{2\pi} \sin x dx = ?$

- A. -2 B. 0 C. -1 D. 2

DO NOT WRITE ANYTHING HERE

- (x) The equation $ax^2 + 2hxy + by^2 = 0$ represents two real and distinct straight lines if:
- A. $h^2 > ab$ B. $h^2 < ab$ C. $h^2 = ab$ D. $h = 0$
- (xi) An equation of the line through $A(-2, -3)$ and $B(4, -6)$ is:
- A. $x + 2y + 8 = 0$ B. $2x + y - 11 = 0$
 C. $2x - y - 3 = 0$ D. $2x + y - 2 = 0$
- (xii) The solution set of inequality $ax + by < c$ is the:
- A. Circle B. Parabola C. Half Plane D. Plane
- (xiii) The feasible solution which maximizes or minimizes the objective function is called:
- A. Feasible solution B. Real solution
 C. Optimal solution D. None of these
- (xiv) If the circle $x^2 + y^2 + x + 2y + c = 0$ passes through $(-2, -1)$ then C is equal to:
- A. -1 B. 1 C. 2 D. -3
- (xv) The vertex of parabola $(x - 1)^2 = 8(y + 2)$ is:
- A. $(1, 2)$ B. $(-1, 2)$ C. $(-1, -2)$ D. $(1, -2)$
- (xvi) Length of latus rectum of parabola $y^2 = 4ax$ is:
- A. a B. $2a$ C. $4a$ D. $\frac{a}{2}$
- (xvii) Vertices of hyperbola $\frac{x^2}{16} - \frac{y^2}{25} = 1$ are:
- A. $(\pm 4, 0)$ B. $(0, \pm 4)$ C. $(0, \pm 5)$ D. $(\pm 5, 0)$
- (xviii) If $\sqrt{3}$ and 1 are x and y components of a vector, then its angle with x -axis is:
- A. 30° B. 45° C. 60° D. 90°
- (xix) The direction cosines of $3\mathbf{i} + 7\mathbf{j} - 4\mathbf{k}$ is:
- A. $\left[\frac{3}{\sqrt{74}}, \frac{7}{\sqrt{74}}, \frac{4}{\sqrt{74}} \right]$ B. $\left[\frac{3}{\sqrt{74}}, \frac{-7}{\sqrt{74}}, \frac{-4}{\sqrt{74}} \right]$
 C. $\left[\frac{3}{\sqrt{74}}, \frac{7}{\sqrt{74}}, \frac{-4}{\sqrt{74}} \right]$ D. $\left[\frac{3}{\sqrt{74}}, \frac{2}{\sqrt{74}}, \frac{-4}{\sqrt{74}} \right]$
- (xx) $(\mathbf{i} + 2\mathbf{j}) \times \mathbf{k} = ?$
- A. $3\mathbf{i} - \mathbf{j}$ B. $2\mathbf{i} - \mathbf{j}$ C. $2 + 2\mathbf{k}$ D. None of these

For Examiner's use only:

Total Marks:

20

Marks Obtained:



MATHEMATICS HSSC-II

Time allowed: 2:35 Hours

Total Marks Sections B and C: 80

NOTE: Attempt any ten parts from Section 'B' and any five questions from Section 'C' on the separately provided answer book. Use supplementary answer sheet i.e. Sheet-B if required. Write your answers neatly and legibly. Graph paper will be provided on demand.

SECTION - B (Marks 40)

Q. 2 Attempt any TEN parts. All parts carry equal marks.

(10 x 4 = 40)

(i) Evaluate $\lim_{x \rightarrow 0} \frac{\sqrt{x+a} - \sqrt{a}}{x}$

(ii) Find the value of m so that the given function is continuous at $x = 3$

$$f(x) = \begin{cases} mx & \text{if } x < 3 \\ x^2 & \text{if } x \geq 3 \end{cases}$$

(iii) If, $y = x^4 + 2x^2 + 2$, prove that $\frac{dy}{dx} = 4x\sqrt{y-1}$

(iv) Differentiate $\text{Log}_a x$ by ab-intio method.

(v) Show that $\cos(x+h) = \cos x - h \sin x - \frac{h^2}{2!} \cos x + \frac{h^3}{3!} \sin x + \dots$

(vi) Evaluate $\int \frac{\sin x + \cos^3 x}{\cos^2 x \sin x} dx$.

(vii) Find the area above the x -axis bounded by the curve $y^2 = 3 - x$ from $x = -1$ to $x = 2$

(viii) Evaluate $\int_0^1 \frac{3x}{\sqrt{4-3x}} dx$

(ix) Find an equation of the parabola with foci $(1, 2)$ and vertex $(3, 2)$.

(x) Find the centre and radius of the circle $x^2 + y^2 - 6x + 4y + 13 = 0$

(xi) Find an equation of the line through $(11, -5)$ and parallel to a line with slope -24 .

(xii) An equation of two parallel lines perpendicular to $2x - y + 3 = 0$ such that the product of x and y intercepts of each is 3

(xiii) Find a and b so that the vectors $3\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ and $a\mathbf{i} + b\mathbf{j} - 2\mathbf{k}$ are parallel.

(xiv) Find the value of the volume of the tetrahedron whose vertices are $A(2,1,8)$, $B(3,2,9)$, $C(2,1,4)$ and $D(3,3,10)$.

SECTION - C (Marks 40)

Note: Attempt any FIVE questions. All questions carry equal marks.

(5 x 8 = 40)

Q. 3 Prove that $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$

Q. 4 If $x = \sin \theta$, $y = \sin m\theta$, show that $(1-x^2)y_2 - xy_1 + m^2y = 0$.

Q. 5 Evaluate $\int_0^{2\pi} \frac{\sin x}{(1+\cos x)(2+\cos x)} dx$

Q. 6 Maximize $Z = 2x + 3y$ subject to the constraints $3x + 4y < 12$, $2x + y \leq 4$, $2x - y \leq 4$, $x \geq 0$, $y \geq 0$.

Q. 7 Find the distance between the given parallel lines, sketch the lines, also find an equation of the parallel lines lying midway between them.

$$\ell_1 : 3x - 4y + 3 = 0$$

$$\ell_2 : 3x - 4y + 7 = 0$$

Q. 8 Find equation of the normal to the parabola $y^2 = 8x$ which are parallel to the line $2x + 3y = 0$.

Q. 9 Find the moment about $A(1,1,1)$ of each of the concurrent forces $\mathbf{i} - 2\mathbf{j}$, $3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, $5\mathbf{j} + 2\mathbf{k}$ where $P(2,0,1)$ is the point of concurrency.

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MATHEMATICS HSSC-II

SECTION – A (Marks 20)

Time allowed: 25 Minutes

NOTE: Section-A is compulsory and comprises pages 1-2. All parts of this section are to be answered on the question paper itself. It should be completed in the first 25 minutes and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. Do not use lead pencil.

Q. 1 Circle the correct option i.e. A / B / C / D. Each part carries one mark.

(i) What is the range of f^{-1} , when $f(x) = 2 + \sqrt{x-1}$?

- A. $[1, \infty)$ B. $(-\infty, -1]$ C. $[-1, 1]$ D. $[2, \infty)$

(ii) $\lim_{x \rightarrow 0} \frac{2-3x}{\sqrt{3+4x^2}} = ?$

- A. $\frac{3}{2}$ B. $-\frac{3}{2}$ C. $\pm \frac{3}{2}$ D. None of these

(iii) $\ln(1+x) = ?$

- A. $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$ B. $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$
C. $x + \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$ D. $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$

(iv) If P is the perimeter of square and A its area then $A = ?$

- A. $\frac{P^2}{4}$ B. $\frac{P^2}{8}$ C. $\frac{P^2}{16}$ D. $16P^2$

(v) $\int \frac{dx}{9-x^2} = ?$

- A. $\frac{1}{6} \ln \left(\frac{x+3}{x-3} \right) + C$ B. $\frac{1}{6} \ln \left(\frac{3+x}{3-x} \right) + C$
C. $\frac{1}{9} \tan^{-1} \frac{x}{9} + C$ D. $\frac{1}{3} \tan^{-1} \frac{x}{3} + C$

(vi) If $f(x) = \frac{2x+1}{2x-1}$ then $f^{-1}(x) = ?$

- A. $\frac{1}{2} \left(\frac{x+1}{x-1} \right)$ B. $\frac{1}{2} \left(\frac{x-1}{x+1} \right)$ C. $\frac{1}{2} \left(\frac{x+2}{x-2} \right)$ D. None of these

(vii) $x = a \sec \theta$, $y = b \tan \theta$ are the parametric equations of:

- A. Ellipse B. Circle C. Hyperbola D. Parabola

(viii) $\frac{d}{dx} \left(\sin \frac{a}{x} \right) = ?$

- A. $-\frac{1}{x^2} \cos \frac{a}{x}$ B. $\frac{1}{x} \cos \frac{a}{x}$ C. $\frac{1}{a} \cos \frac{a}{x}$ D. None of these

(ix) $\int_{-1}^3 (x^3 + 3x^2) dx = ?$

- A. 28 B. 48 C. 58 D. 20

- (x) Distance of the point (x, y) from x-axis is:
 A. x B. y C. $|x|$ D. $|y|$
- (xi) The slope of the line $2x + 3y = 7$ is:
 A. $\frac{2}{3}$ B. $\frac{1}{3}$ C. $-\frac{2}{3}$ D. $-\frac{1}{2}$
- (xii) The coordinates of the point that divides the join of A $(-6, 3)$ and B $(5, -2)$ in the ratio $2 : 3$, internally is:
 A. $\left(1, \frac{8}{5}\right)$ B. $\left(\frac{-8}{5}, 1\right)$ C. $\left(0, \frac{8}{5}\right)$ D. None of these
- (xiii) The two lines ℓ_1 and ℓ_2 with respective slope m_1 and m_2 are perpendicular if:
 A. $m_1 + m_2 = 0$ B. $m_1 m_2 = 1$ C. $m_1 m_2 = -1$ D. $m_1 - m_2 = 0$
- (xiv) A region which is restricted to the first quadrant is called:
 A. Maximum region B. Minimum region
 C. Feasible region D. Objective function
- (xv) The centre of a circle $x^2 + y^2 + 6x - 10y - 15 = 0$ is:
 A. $(5, 3)$ B. $(5, -3)$ C. $(-3, 5)$ D. $(3, 5)$
- (xvi) If $e < 1$, then the conic is called:
 A. Parabola B. Circle C. Hyperbola D. Ellipse
- (xvii) The focus of a parabola $x^2 = -16y$ is:
 A. $(-4, 0)$ B. $(0, -4)$ C. $(4, -4)$ D. $(0, \pm 4)$
- (xviii) If $\underline{A} = \underline{i} + \sqrt{3}\underline{j}$, then the unit vector \hat{A} is:
 A. $\frac{-\underline{i} + \sqrt{3}\underline{j}}{2}$ B. $\frac{\underline{i} + \sqrt{3}\underline{j}}{2}$
 C. $\frac{\underline{i} - \sqrt{3}\underline{j}}{2}$ D. None of these
- (xix) A vector perpendicular to $2\underline{i} - \underline{j} + \underline{k}$ and $4\underline{i} + 2\underline{j} + 8\underline{k}$ is:
 A. $-\underline{i} + 6\underline{j} + 8\underline{k}$ B. $-10\underline{i} - 12\underline{j} + 8\underline{k}$
 C. $\underline{i} + 6\underline{j} - 8\underline{k}$ D. $-\underline{i} + 6\underline{j} - 8\underline{k}$
- (xx) $(2\underline{i}, 3\underline{j}) \times \underline{k}$ is:
 A. $2\underline{i} - \underline{j}$ B. $2\underline{i} - 3\underline{k}$
 C. 0 D. None of these

For Examiner's use only:

Total Marks:

20

Marks Obtained:



MATHEMATICS HSSC-II

31

Time allowed: 2:35 Hours

Total Marks Sections B and C: 80

NOTE: Attempt any ten parts from Section 'B' and any five questions from Section 'C' on the separately provided answer book. Use supplementary answer sheet i.e. Sheet-B if required. Write your answers neatly and legibly. Graph paper will be provided on demand.

SECTION - B (Marks 40)

Q. 2 Attempt any TEN parts. All parts carry equal marks.

(10 x 4 = 40)

- (i) Evaluate $\lim_{x \rightarrow 0} \frac{\sin x^0}{x}$
- (ii) Graph the curves that has parametric equations given below:
 $x = t - 1, y = 2t - 1 \quad -1 < t < 5$ Where 't' is a parameter
- (iii) Prove that $y \frac{dy}{dx} + x = 0$ if $x = \frac{1-t^2}{1+t^2}, y = \frac{2t}{1+t^2}$
- (iv) If $y = \tan(P \tan^{-1} x)$, show that $(1+x^2)y_1 - P(1+y^2) = 0$
- (v) Find the point on the curve $y = x^2 + 1$ that is closest to the point (18, 1).
- (vi) Evaluate $\int \frac{x^2}{3+x^2} dx$
- (vii) Evaluate $\int \left(\frac{1 - \sin x}{1 - \cos x} \right) e^x dx$
- (viii) Evaluate $\int_0^{\frac{\pi}{4}} \sec x (\sec x + \tan x) dx$.
- (ix) Find an equation of the line through (5, -8) and perpendicular to the joint of A(-15, -8) and B (10,7).
- (x) Find the point which is equidistant from the points A (5, 3), B (-2,2) and C (4, 2).
- (xi) Find the centre and radius of the circle with the given equation $4x^2 + 4y^2 - 8x + 12y - 25 = 0$.
- (xii) Find an equation of the ellipse with foci $(\pm 3, 0)$ and minor axis of length 10.
- (xiii) A parabolic arch has a 100 m base and height 25m. Find the height of arch at the point 30 m from the centre of the base.
- (xiv) Prove that altitudes of a triangle are concurrent.

SECTION - C (Marks 40)

Note: Attempt any FIVE questions. All questions carry equal marks.

(5 x 8 = 40)

Q. 3 If $f(x) = \begin{cases} 3x & \text{if } x \leq -2 \\ x^2 - 1 & \text{if } -2 < x < 2 \\ 3 & \text{if } x \geq 2 \end{cases}$

Discuss the continuity at $x=2$ and $x=-2$.

Q. 4 Solve the differential equation $y - x \frac{dy}{dx} = 3 \left(1 + x \frac{dy}{dx} \right)$

Q. 5 If $y = a \cos(l nx) + b \sin(l nx)$, prove that $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$

Q. 6 Find the area of the region bounded by the triangle whose sides are $x - 2y - 6 = 0, 3x - y + 3 = 0, 2x + y - 4 = 0$

Q. 7 Maximize the function defined as $f(x, y) = 2x + 3y$, subject to the constraints $2x + y \leq 8, x + 2y \leq 14, x \geq 0, y \geq 0$

Q. 8 Find the equation of the tangents to the ellipse $\frac{x^2}{128} + \frac{y^2}{8} = 1$

Which are parallel to the line $3x + 8y + 1 = 0$, also find the points of contact.

Q. 9 A particle displaced from the point A (5, -5, -7) to the point B (6, 2, -2) under the action of constant forces defined by $10\mathbf{i} - \mathbf{j} + 11\mathbf{k}, 4\mathbf{i} + 5\mathbf{j} + 9\mathbf{k}$ and $-2\mathbf{i} + \mathbf{j} - 9\mathbf{k}$. Show that the total work done by the forces is 102 units.