Answer Key to the Model Question Paper (Class – XI)

Section – A (Marks 20)

Question # 01 (MCQs)

Parts	Correct Option	Parts	Correct Option
1.	(A)	11.	(A)
2.	(A)	12.	(B)
3.	(C)	13.	(C)
4.	(A)	14.	(A)
5.	(A)	15.	(B)
6.	(B)	16.	(B)
7.	(B)	17.	(A)
8.	(D)	18.	(A)
9.	(A)	19.	? ? ?
10.	(C)	20.	(C)

Section – B (Marks 48)

Q2(i):

Solution: $z_1 = 1 - 2i,\ z_2 = 2 + 3i,\ z_3 = 4 - 3i$

(a) $\frac{\overline{z_2}}{z_3} = \frac{\overline{2+3i}}{4-3i} = \frac{2-3i}{4-3i}$

$\Rightarrow \frac{2-3i}{4-3i} \times \frac{4+3i}{4+3i} = \frac{(2-3i)(4+3i)}{(4-3i)(4-3i)} = \frac{8+6i-12i-9i^2}{16-9i^2}$

Since , $i^2 = -1$

So, $\frac{\overline{z_2}}{z_3} = \frac{8-6i+9}{16+9} = \frac{17}{25} - \frac{6}{25}i$

(b) $\overline{z_1} \cdot \overline{z_3} = (\overline{1-2i}) \cdot (\overline{4-3i})$

$\Rightarrow \overline{z_1} \cdot \overline{z_3} = (1+2i)(4+3i) = 4 + 3i + 8i + 6i^2 = 4 + 11i - 6 \qquad \because i^2 = -1$

$\Rightarrow \overline{z_1} \cdot \overline{z_3} = -2 + 11i$

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Q2(ii):

Solution: $A \cap B = B \cap A$

We first convert it into the logical form as: $p \wedge q = q \wedge p$

Now, we construct a truth table to prove this equality.

		L.H.S.	**R.H.S.**
p	q	$p \wedge q$	$q \wedge p$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

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Q2(iii):

Solution: $\begin{vmatrix} x & 1 & x+1 \\ 2 & x & 3 \\ x+1 & 4 & x \end{vmatrix} = 11 - 2x^2$

To find the value of x, we expand L.H.S for first row

$\Rightarrow \quad x\begin{vmatrix} x & 3 \\ 4 & x \end{vmatrix} - 1\begin{vmatrix} 2 & 3 \\ x+1 & x \end{vmatrix} + (x+1)\begin{vmatrix} 2 & x \\ x+1 & 4 \end{vmatrix} = 11 - 2x^2$

$\Rightarrow \quad x(x^2 - 12) - (2x - 3x - 3) + (x+1)(8 - x^2 - x) = 11 - 2x^2$

$\Rightarrow \quad x^3 - 12x + x + 3 + 8x - x^3 - x^2 + 8 - x^2 - x = 11 - 2x^2$

$\Rightarrow \quad -2x^2 - 12x + 11 + 2x^2 - 11 = 0$

$\Rightarrow \quad -12x = 0$

$\Rightarrow \quad x = 0$

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Q2(iv):

Solution: $\frac{18}{x^4} + \frac{1}{x^2} = 4$

$\Rightarrow \frac{18 + x^2}{x^4} = 4$

$\Rightarrow \ 18 + x^2 = 4x^4$

OR

$\Rightarrow \ 4x^4 - x^2 - 18 = 0$

$\Rightarrow \ 4x^4 - 9x^2 + 8x^2 - 18 = 0$

$\Rightarrow \ x^2(4x^2 - 9) + 2(4x^2 - 9) = 0$

$\Rightarrow \ (4x^2 - 9)(x^2 + 2) = 0$

Here, $4x^2 - 9 = 0$ and $x^2 + 2 = 0$

$\Rightarrow \ x^2 = \frac{9}{4}$ and $x^2 = -2$

Taking square-root on bothsides

$\Rightarrow \ x = \pm\frac{3}{2}$ and $x = \pm\sqrt{2}i$ (Complex)

Solution Set = $\{\pm\frac{3}{2}\}$

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Q2(v):

Solution: Resolve $\frac{3x^2+7x+28}{x(x^2+x+7)}$ into Partial Fraction

$\Rightarrow \frac{3x^2+7x+28}{x(x^2+x+7)} = \frac{A}{x} + \frac{Bx+C}{x^2+x+7}$ (i)

Multiplying by $x(x^2 + x + 7)$

$\Rightarrow 3x^2 + 7x + 28 = A(x^2 + x + 7) + (Bx + C)(x)$ (ii)

For $x = 0$, Eq. (ii) gives

$\Rightarrow 3(0)^2 + 7(0) + 28 = A(0^2 + 0 + 7) + (B(0) + C)(0)$

$\Rightarrow 28 = 7A, \quad \Rightarrow A = 4$

Expanding Eq. (ii), we get

$\Rightarrow 3x^2 + 7x + 28 = Ax^2 + Ax + 7A + Bx^2 + Cx$

Equating the coefficients of;

$x^2; \quad 3 = A + B, \quad \Rightarrow 4 + B = 3, \quad \Rightarrow B = -1$

$x; \quad 7 = A + C, \quad \Rightarrow 4 + C = 7, \quad \Rightarrow C = 3$

Substituting the values of $A, B, and\ C$ in Eq. (i)

$\Rightarrow \frac{3x^2+7x+28}{x(x^2+x+7)} = \frac{4}{x} + \frac{(-1)x+3}{x^2+x+7}$

$\Rightarrow \frac{3x^2+7x+28}{x(x^2+x+7)} = \frac{4}{x} - \frac{x-3}{x^2+x+7}$

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Q2(vi):

Solution: Since, $f(x + p) = f(x)$

Where, p is called the period of the function.

For Cosine function, we check the periodic function for $p = 2\pi$,

(i.e)

$\cos(x + 2\pi) = cosx \cos(2\pi) - sinx \sin(2\pi)$ (By Fundamental law of trigonometry)

Here, $\cos(2\pi) = 1$, and $\sin(2\pi) = 0$

$\Rightarrow \cos(x + 2\pi) = cosx\ (1) - sinx\ (0)$

$\Rightarrow \quad \cos(x + 2\pi) = cosx$

Hence, the period of Cosine is 2π.

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Q2(vii):

Solution: Sum of an A.P. is;

$S_n = \frac{n}{2}[2a_1 + (n-1)d]$

Sum of first 30 $-$terms of an A.P. is;

$\Rightarrow \quad S_{30} = \frac{30}{2}[2a + (30-1)d]$

$\Rightarrow \quad S_{30} = 15[2a + 29d]$

Similarly, the sum of first 6 $-$terms is;

$\Rightarrow \quad S_6 = 3[2a + 5d]$

The square of sum of first 6 $-$terms will be;

$\Rightarrow \quad S_6^2 = 9(2a + 5d)^2 = 9(4a^2 + 20ad + 25d^2)$

Since, $S_{30} = S_6^2$

$\Rightarrow \quad 15(2a + 29d) = 9(4a^2 + 20ad + 25d^2)$

$\Rightarrow \quad 5(2a + 29d) = 3(4a^2 + 20ad + 25d^2)$

$\Rightarrow \quad 10a + 145d = 12a^2 + 60ad + 75d^2$ (Proved)

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Q2(viii):

Solution:

Number of boys = 8

Number of girls = 6

Person chosen = 4

The cases for atleast one girl are;

Case – 1: 1 girl + 3 boys $= \binom{6}{1} \times \binom{8}{3}$

Case – 2: 2 girls + 2 boys $= \binom{6}{2} \times \binom{8}{2}$

Case – 3: 3 girls + 1 boy $= \binom{6}{3} \times \binom{8}{1}$

Case – 4: 4 girls + 0 boy $= \binom{6}{4} \times \binom{8}{0}$

Probability of more girls than boys is;

$$= \frac{\binom{6}{3}\times\binom{8}{1}}{\binom{14}{4}} + \frac{\binom{6}{4}\times\binom{8}{0}}{\binom{14}{4}} = \frac{160}{1001} + \frac{15}{1001} = \frac{175}{1001} = \frac{25}{143}$$

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Q2(ix):

Solution: There are 7 places to fill;

3 places of alphabet (out of 26) can be filled in $= 26 \times 25 \times 24 = 15600$

4 places of digits (out of 10 – (0 to 9)) can be filled in $= 10 \times 9 \times 8 \times 7 = 5040$

Number of different plates $= 15600 \times 5040 = 78624000$

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Q2(x):

Solution: $\frac{1}{\sqrt{9+x}} = (9+x)^{-\frac{1}{2}} = 9^{-\frac{1}{2}}\left(1+\frac{x}{9}\right)^{-\frac{1}{2}}$

$\Rightarrow \quad 3^{-1}\left(1+\frac{x}{9}\right)^{-\frac{1}{2}} = \frac{1}{3}\left(1+\frac{x}{9}\right)^{-\frac{1}{2}}$

Expanding by Binomial Series

$$\Rightarrow \frac{1}{3}\left\{1 + \left(-\frac{1}{2}\right)\left(\frac{x}{9}\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!}\left(\frac{x}{9}\right)^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)\left(-\frac{1}{2}-2\right)}{3!}\left(\frac{x}{9}\right)^3 + \cdots\right\}$$

$$\Rightarrow \frac{1}{3}\left\{1 - \frac{x}{18} + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2\cdot1}\cdot\frac{x^2}{81} + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3\cdot2\cdot1}\cdot\frac{x^3}{729} + \cdots\right\}$$

$$\Rightarrow \frac{1}{3}\left\{1 - \frac{x}{18} + \frac{1}{8}\cdot\frac{x^2}{27} - \frac{5}{16}\cdot\frac{x^3}{729} + \cdots\right\}$$

$$\Rightarrow \frac{1}{3}\left\{1 - \frac{x}{18} + \frac{x^2}{216} - \frac{5x^3}{11664} + \cdots\right\} = \frac{1}{3} - \frac{x}{54} + \frac{x^2}{648} - \frac{5x^3}{34992} + \cdots$$

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Q2(xi):

Solution: Radius $= 15cm$

Interior angle $= \frac{360^o}{5} = 72^o$

Using law of cosine, we obtain

$\Rightarrow \quad c^2 = a^2 + b^2 - 2abcos\gamma$

$\Rightarrow \quad c^2 = (15)^2 + (15)^2 - 2(15)(15)cos72^o$

$\Rightarrow \quad c^2 = 225 + 225 - 450(0.309)$

$\Rightarrow \quad c^2 = 310.95$

Taking square-root

$\Rightarrow \quad c = 17.63cm$

Perimeter of Pentagon $= 5(17.63) = 88.17cm$

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Q2(xii):

Solution: $\frac{sin3\theta+sin5\theta+sin7\theta}{cos3\theta+cos5\theta+cos7\theta} = tan5\theta$

Taking, $L.H.S. = \frac{sin3\theta+sin7\theta+sin5\theta}{cos3\theta+cos7\theta+cos5\theta}$

$\Rightarrow \quad L.H.S. = \frac{2\sin\left(\frac{10\theta}{2}\right)\cos\left(\frac{4\theta}{2}\right)+sin5\theta}{2\cos\left(\frac{10\theta}{2}\right)\cos\left(\frac{4\theta}{2}\right)+cos5\theta}$

$\Rightarrow \quad L.H.S. = \frac{2\sin 5\theta\cos 2\theta+sin5\theta}{2\cos 5\theta\cos 2\theta+cos5\theta} = \frac{sin5\theta(2cos2\theta+1)}{cos5\theta(2\cos 2\theta+1)}$

$\Rightarrow \quad L.H.S. = \frac{sin5\theta}{cos5\theta} = tan5\theta = R.H.S.$ (Proved)

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Q2(xiii):

Solution: $y = sec2x$; $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

x	$-\pi/2$	$-\pi/3$	$-\pi/6$	0	$\pi/6$	$\pi/3$	$\pi/2$
$y = sec2x$	-1	-2	2	1	2	-2	-1
$P(x,y)$	$\left(-\frac{\pi}{2},-1\right)$	$\left(-\frac{\pi}{3},-2\right)$	$\left(-\frac{\pi}{6},2\right)$	$(0,1)$	$\left(\frac{\pi}{6},2\right)$	$\left(\frac{\pi}{3},-2\right)$	$\left(\frac{\pi}{2},-1\right)$

Plot the graph on graph paper using the values calculated in the above table.

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Q2(xiv):

Solution: $s^2 = \triangle \cot\left(\frac{\alpha}{2}\right)\cot\left(\frac{\beta}{2}\right)\cot\left(\frac{\gamma}{2}\right)$

Taking, $R.H.S. = \triangle \cot\left(\frac{\alpha}{2}\right)\cot\left(\frac{\beta}{2}\right)\cot\left(\frac{\gamma}{2}\right)$

Where, $\cot\left(\frac{\alpha}{2}\right) = \sqrt{\frac{s(s-a)}{(s-b)(s-c)}}$; $\cot\left(\frac{\beta}{2}\right) = \sqrt{\frac{s(s-b)}{(s-c)(s-a)}}$; $\cot\left(\frac{\gamma}{2}\right) = \sqrt{\frac{s(s-c)}{(s-a)(s-b)}}$

$\Rightarrow \quad R.H.S. = \triangle \sqrt{\frac{s(s-a)}{(s-b)(s-c)} \cdot \frac{s(s-b)}{(s-c)(s-a)} \cdot \frac{s(s-c)}{(s-a)(s-b)}}$

$\Rightarrow \quad R.H.S. = \triangle \sqrt{\frac{s^2 \cdot s(s-a)(s-b)(s-c)}{(s-a)^2(s-b)^2(s-c)^2}} = \triangle \sqrt{\frac{s^2 \cdot s^2}{s(s-a)(s-b)(s-c)}}$

$\Rightarrow \quad R.H.S. = \triangle \left(\frac{s^2}{\triangle}\right) = s^2 = L.H.S.$ (Proved)

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Q2(xv):

Solution: $\cot^{-1}\left(\frac{119}{120}\right) = 2\sin^{-1}\left(\frac{5}{13}\right)$

Let $y = \cot^{-1}\left(\frac{119}{120}\right)$

$\Rightarrow \quad coty = \frac{119}{120}$

Where, $cscy = \sqrt{1 + \cot^2 y} = \sqrt{1 + \frac{14161}{14400}} = \sqrt{\frac{28561}{14400}}$

$\Rightarrow \quad cscy = \frac{169}{120}$

$\Rightarrow \quad siny = \frac{120}{169}$

Here, $cosy = \sqrt{1 - \sin^2 y} = \sqrt{1 - \frac{14400}{28561}} = \frac{119}{169}$

Using Half angle identity

$\Rightarrow \quad \sin\left(\frac{y}{2}\right) = \sqrt{\frac{1-cosy}{2}} = \sqrt{\frac{1}{2}\left(1 - \frac{119}{169}\right)} = \sqrt{\frac{25}{169}}$

$\Rightarrow \quad \sin\left(\frac{y}{2}\right) = \frac{5}{13}$; $\Rightarrow \quad y = 2\sin^{-1}\left(\frac{5}{13}\right)$

Hence, $\cot^{-1}\left(\frac{119}{120}\right) = 2\sin^{-1}\left(\frac{5}{13}\right)$ Proved

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Q2(xvi):

Solution: We shall show that $2\cos^2\theta + 3\cos\theta - 2 = 0$

Where, $2\tan^2\theta cos\theta = 3$

$\Rightarrow \quad 2\left(\frac{\sin^2\theta}{\cos^2\theta}\right) cos\theta = 3$

$\Rightarrow \quad 2(1 - \cos^2\theta) = 3cos\theta$

$\Rightarrow \quad 2 - 2\cos^2\theta = 3cos\theta$

$\Rightarrow \quad 2\cos^2\theta + 3cos\theta - 2 = 0$ (Proved)

Also, to find the solution, we take $2\cos^2\theta + 3cos\theta - 2 = 0$

$\Rightarrow \quad 2\cos^2\theta + 4cos\theta - cos\theta - 2 = 0$

$\Rightarrow \quad 2\,cos\,\theta(cos\theta + 2) - 1(cos\theta + 2) = 0$

$\Rightarrow \quad (cos\theta + 2)(2cos\theta - 1) = 0$

Here, $cos\theta + 2 = 0$ and $2cos\theta - 1 = 0$

$\Rightarrow \quad cos\theta = -2$ (Not possible) and $\Rightarrow \quad cos\theta = \frac{1}{2}$

$\Rightarrow \quad \theta = \frac{\pi}{3}$ and $\Rightarrow \quad \theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

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Section – C (Marks 32)

Q3.

Solution: $x + 2y + 3z = 3 \;\; ; \;\; 2x + 3y + z = 1 \;\; ; \;\; 3x + y + 2z = 2$

Let A_b be the augmented matrix, then

$$\Rightarrow \quad A_b = \begin{bmatrix} 1 & 2 & 3 & \vdots & 3 \\ 2 & 3 & 1 & \vdots & 1 \\ 3 & 1 & 2 & \vdots & 2 \end{bmatrix}$$

To reduce A_b into reduced echelon form by elementary Row operations

$$\Rightarrow \quad R_2 - (2)R_1 \;, R_3 - (3)R_1 \;\sim^R \begin{bmatrix} 1 & 2 & 3 & \vdots & 3 \\ 0 & -1 & -5 & \vdots & -5 \\ 0 & -5 & -7 & \vdots & -7 \end{bmatrix}$$

$$\Rightarrow \quad (-1)R_2 \;\sim^R \begin{bmatrix} 1 & 2 & 3 & \vdots & 3 \\ 0 & 1 & 5 & \vdots & 5 \\ 0 & -5 & -7 & \vdots & -7 \end{bmatrix}$$

$$\Rightarrow \quad R_1 - (2)R_2 \;, R_3 + (5)R_2 \;\sim^R \begin{bmatrix} 1 & 0 & -7 & \vdots & -7 \\ 0 & 1 & 5 & \vdots & 5 \\ 0 & 0 & 18 & \vdots & 18 \end{bmatrix}$$

$$\Rightarrow \quad \left(\frac{1}{18}\right) R_3 \;\sim^R \begin{bmatrix} 1 & 0 & -7 & \vdots & -7 \\ 0 & 1 & 5 & \vdots & 5 \\ 0 & 0 & 1 & \vdots & 1 \end{bmatrix}$$

$$\Rightarrow \quad R_1 + (7)R_3 \;, R_2 - (5)R_3 \;\sim^R \begin{bmatrix} 1 & 0 & 0 & \vdots & 0 \\ 0 & 1 & 0 & \vdots & 0 \\ 0 & 0 & 1 & \vdots & 0 \end{bmatrix}$$

$\therefore \quad x = 0 \;, \; y = 0 \;, \; z = 0$

Hence, the solution is trivial, (i.e.) $\{(0,0,0)\}$

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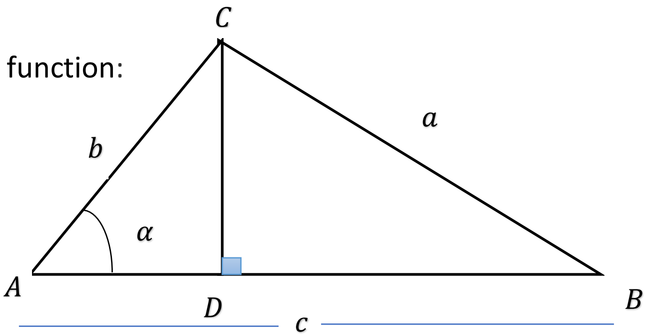
Q4.

Proof (a): $a^2 = b^2 + c^2 - 2bc\ cos\alpha$



In the right triangle ACD, by the definition of cosine function:

$cos\ \alpha = \frac{AD}{b} \Rightarrow AD = b\ cos\alpha$ (i)

$\Rightarrow DB = c - b\ cos\alpha$ (ii)

In the triangle ACD, according to Sine definition

$sin\ \alpha = \frac{CD}{b} \Rightarrow CD = b\ sin\alpha$ (iii)

In the triangle BDC, if we apply the Pythagorean Theorem, then

$a^2 = (BD)^2 + (CD)^2$ (iv)

Substituting for BD and CD from equations (ii) and (iii) in equation (iv)

$\Rightarrow a^2 = (c - b\ cos\alpha)^2 + (b\ sin\alpha)^2$

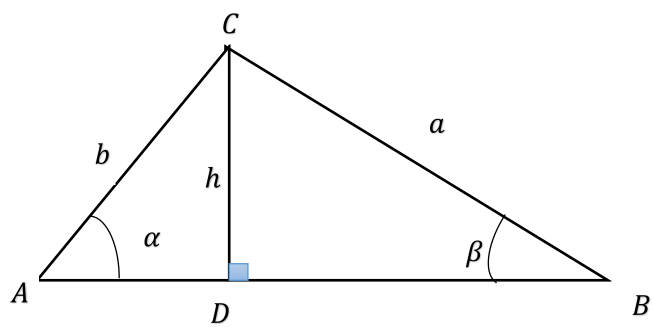
$\Rightarrow a^2 = c^2 - 2bc\ cos\alpha + b^2\cos^2\alpha + b^2\sin^2\alpha$

$\Rightarrow a^2 = b^2(\cos^2\alpha + \sin^2\alpha) + c^2 - 2bc\ cos\alpha$

Where, $\cos^2\alpha + \sin^2\alpha = 1$

$\Rightarrow a^2 = b^2 + c^2 - 2bc\ cos\alpha$ (Proved)

Proof (b): $\frac{a}{sin\alpha} = \frac{b}{sin\beta} = \frac{c}{sin\gamma}$



In right-angled triangle ADC

$sin\alpha = \frac{h}{b} \Rightarrow h = b\ sin\alpha$ (i)

In right-angled triangle BDC,

$sin\beta = \frac{h}{a} \Rightarrow h = a\ sin\beta$ (ii)

Comparing Equations (i) and (ii), we get

$a\ sin\beta = b\ sin\alpha$

$\Rightarrow \frac{a}{sin\alpha} = \frac{b}{sin\beta}$ (iii)

Similarly, it can also be proved that $\frac{b}{sin\beta} = \frac{c}{sin\gamma}$ or $\frac{c}{sin\gamma} = \frac{a}{sin\alpha}$

Thus, $\frac{a}{sin\alpha} = \frac{b}{sin\beta} = \frac{c}{sin\gamma}$ (Proved)

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Q5.

Solution:

(a) nth term of the series is;

$$a_n = \frac{4}{5}\left(\frac{2}{3}\right)^{n-1}$$

For $n = 1$

$$\Rightarrow \quad a_1 = \frac{4}{5}\left(\frac{2}{3}\right)^{1-1} = \frac{4}{5}\left(\frac{2}{3}\right)^{0}$$

$$\Rightarrow \quad a_1 = \frac{4}{5}$$

For $n = 2$

$$\Rightarrow \quad a_2 = \frac{4}{5}\left(\frac{2}{3}\right)^{2-1} = \frac{4}{5}\left(\frac{2}{3}\right)^{1}$$

$$\Rightarrow \quad a_2 = \frac{8}{15}$$

For $n = 3$

$$\Rightarrow \quad a_3 = \frac{4}{5}\left(\frac{2}{3}\right)^{3-1} = \frac{4}{5}\left(\frac{2}{3}\right)^{2}$$

$$\Rightarrow \quad a_3 = \frac{16}{45}$$

$\therefore \quad \frac{4}{5} + \frac{8}{15} + \frac{16}{45} + \cdots$, the Geometric series with Common Ratio $= \frac{2}{3}$

(b) To find the sum of first ten terms of the geometric series, we have

$$S_n = \frac{a_1(1-r)^n}{1-r}$$

Here, $a_1 = \frac{4}{5}, \quad r = \frac{2}{3} < 1, n = 10$

$$\Rightarrow \quad S_{10} = \frac{\frac{4}{5}\left(1-\frac{2}{3}\right)^{10}}{1-\frac{2}{3}} = \frac{4}{5}\left(\frac{1}{3}\right)^{10} \times 3$$

$\Rightarrow \quad S_{10} = \frac{4}{5\times 3^9} = \frac{4}{98415}$

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Q6.

Solution: We shall prove that $63y^2+84y+19=0$

Where, $y=-\frac{1}{3}+\frac{1}{3^3}+\frac{1.3}{2!}.\frac{1}{3^5}+\frac{1.3.5}{3!}.\frac{1}{3^7}+\cdots$

$$\Rightarrow \quad y+\frac{1}{3}=\frac{1}{3}\left(\frac{1}{3^2}+\frac{1.3}{2!}.\frac{1}{3^4}+\frac{1.3.5}{3!}.\frac{1}{3^6}+\cdots\right)$$

$$\Rightarrow \quad 3y+1+1=1+\frac{1}{3^2}+\frac{1.3}{2!}.\frac{1}{3^4}+\frac{1.3.5}{3!}.\frac{1}{3^6}+\cdots$$

$\Rightarrow \quad 3y+2=1+\frac{1}{3^2}+\frac{1.3}{2!}.\frac{1}{3^4}+\frac{1.3.5}{3!}.\frac{1}{3^6}+\cdots$ (A)

Let the R.H.S. of the series be identical as;

$(1+x)^n=1+nx+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3+\cdots$ (B)

Equating the second and third terms on R.H.S. of above equations, we get

$nx=\frac{1}{3^2}=\frac{1}{9}$ (i)

$\frac{n(n-1)}{2!}x^2=\frac{1.3}{2!}.\frac{1}{3^4}$ (ii)

From Eq. (i); $x=\frac{1}{3^2 n}$ (iii)

Substituting Eq. (iii) in Eq. (ii)

$$\frac{n(n-1)}{2!}\left(\frac{1}{3^2 n}\right)^2=\frac{1.3}{2!}.\frac{1}{3^4}$$

$$\Rightarrow \quad \frac{n(n-1)}{n^2}=\frac{1.3}{2!}\times\frac{1}{3^4}\times 2!\times 3^4$$

$$\Rightarrow \quad n-1=3n$$

$$\Rightarrow \quad n=-\frac{1}{2}$$

Utilizing the value of n in Eq. (iii), it gives

$$x=-\frac{2}{9}$$

Now, substituting the values of x and n in the L.H.S. of Eqs. (A) and (B), we obtain

$$3y+2=\left(1-\frac{2}{9}\right)^{-\frac{1}{2}}$$

$$\Rightarrow \quad 3y + 2 = \left(\frac{9}{7}\right)^{\frac{1}{2}}$$

Squaring the bothsides

$$\Rightarrow \quad (3y+2)^2 = \frac{9}{7}$$

$$\Rightarrow \quad 7(9y^2 + 12y + 4) = 9$$

$$\Rightarrow \quad 63y^2 + 84y + 19 = 0$$

Hence, proved.

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Q7.

Solution: $\cos\frac{\pi}{18} . \cos\frac{\pi}{6} . \cos\frac{5\pi}{18} . \cos\frac{7\pi}{18} = \frac{3}{16}$

We take;

$$L.H.S. = \cos\frac{\pi}{18} . \cos\frac{\pi}{6} . \cos\frac{5\pi}{18} . \cos\frac{7\pi}{18}$$

$$L.H.S. = \cos\frac{\pi}{6} . \frac{1}{2}\left(2\cos\frac{5\pi}{18} . \cos\frac{\pi}{18}\right) . \cos\frac{7\pi}{18}$$

Where, $2cos\alpha\ cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$

$\therefore$

$$L.H.S. = \frac{\sqrt{3}}{4}\left(\cos\left(\frac{6\pi}{18}\right) + \cos\left(\frac{4\pi}{18}\right)\right) . \cos\frac{7\pi}{18}$$

$$L.H.S. = \left(\frac{\sqrt{3}}{8} + \frac{\sqrt{3}}{4}\cos\left(\frac{2\pi}{9}\right)\right) . \cos\frac{7\pi}{18}$$

$$L.H.S. = \frac{\sqrt{3}}{8} . \cos\frac{7\pi}{18} + \frac{\sqrt{3}}{8}\left(2\cos\left(\frac{2\pi}{9}\right) . \cos\left(\frac{7\pi}{18}\right)\right)$$

$$L.H.S. = \frac{\sqrt{3}}{8} . \cos\frac{7\pi}{18} + \frac{\sqrt{3}}{8}\left(\cos\left(\frac{11\pi}{18}\right) + \cos\left(\frac{3\pi}{18}\right)\right)$$

$$L.H.S. = \frac{\sqrt{3}}{8} . \cos\frac{7\pi}{18} + \frac{\sqrt{3}}{8} . \cos\frac{11\pi}{18} + \frac{\sqrt{3}}{8} . \frac{\sqrt{3}}{2}$$

$$L.H.S. = \frac{\sqrt{3}}{8}\left\{\left(cos\frac{7\pi}{18}\right) + \left(cos\frac{11\pi}{18}\right)\right\} + \frac{3}{16}$$

$$L.H.S. = \frac{\sqrt{3}}{8}\left(2\cos\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{9}\right)\right) + \frac{3}{16}$$

$$L.H.S. = \frac{3}{16} = R.H.S.$$

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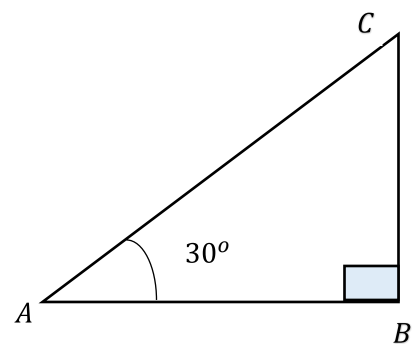
Q8.

Solution: ABC is a right-angled triangle with $m\angle B = 90^o$, $m\angle A = 30^o$ and $m\overline{AB} = 3cm$



(a) First, we calculate $m\angle C$ by

$$m\angle A + m\angle B + m\angle C = 180^o$$

$$\Rightarrow \quad 30^o + 90^o + m\angle C = 180^o$$

$$\Rightarrow \quad m\angle C = 180^o - 120^o$$

$$\Rightarrow \quad m\angle C = 60^o$$

Second, $cos30^o = \frac{3}{m\overline{AC}} \quad \Rightarrow \quad m\overline{AC} = 2\sqrt{3}cm$

Third, $sin30^o = \frac{m\overline{BC}}{2\sqrt{3}} \quad \Rightarrow \quad m\overline{BC} = \sqrt{3}cm$

(b) Area of triangle $(\Delta ABC) = \frac{1}{2}(m\overline{AB})(m\overline{BC})$

$$\Rightarrow \quad \Delta = \frac{1}{2}(3)(\sqrt{3})$$

$$\Rightarrow \quad \Delta = \frac{3\sqrt{3}}{2}\ cm^2$$

(c) Radius of circum-circle $(R) = \frac{abc}{4\Delta}$

$$\Rightarrow \quad R = \frac{(3)(2\sqrt{3})(\sqrt{3})}{4\left(\frac{3\sqrt{3}}{2}\right)} = \sqrt{3}cm$$

(d) Radius of in-circle $(r) = \frac{\Delta}{s}$

Where, $s = \frac{a+b+c}{2} = \frac{3+2\sqrt{3}+\sqrt{3}}{2} = \frac{3(1+\sqrt{3})}{2}$

$$\Rightarrow \quad r = \frac{\left(\frac{3\sqrt{3}}{2}\right)}{\left(\frac{3(1+\sqrt{3})}{2}\right)}$$

$$\Rightarrow \quad r = \frac{\sqrt{3}}{1+\sqrt{3}}$$

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